

Looking Ahead to Chapter 2

Focus In Chapter 2, you will learn how to find the volume and surface area of pyramids, prisms, cylinders, cones, and spheres.

Chapter Warmup

Answer these questions to help you review skills that you will need in Chapter 2.

Find each product.

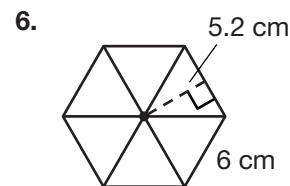
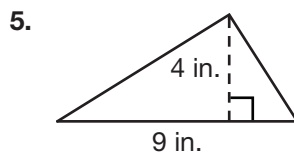
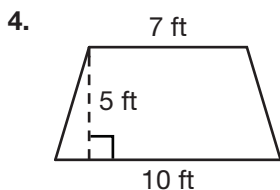
2

1. $8 \cdot 2 \cdot 3$

2. $(4)(5)(7)$

3. $5 \cdot 5 \cdot 5$

Find the area of each polygon.



Read the problem scenario below.

You and your friends decide to get together and make pizza for dinner. Some of your friends prefer circular pizzas, and some of your friends prefer rectangular pizzas. You have 2 different kinds of pizza pans. One is a rectangular pizza pan that is 12 inches long and 10 inches wide. Another pizza pan is circular with a diameter of 14 inches.

7. Which pizza pan has the larger perimeter?

8. Which pizza pan has the larger area?

Key Terms

solid ■ p. 61

volume ■ p. 61, 69, 93, 98, 105

surface area ■ p. 62

polyhedron ■ p. 65

faces ■ p. 65

prism ■ p. 66

bases ■ p. 66

lateral faces ■ p. 66, 80

height ■ p. 66, 81, 92, 97

lateral area ■ p. 74, 86

surface area ■ p. 74, 87, 96,
101, 107

pyramid ■ p. 80

vertex ■ p. 80

regular pyramid ■ p. 81

slant height ■ p. 81, 97

composite solid ■ p. 88

cylinder ■ p. 92

cone ■ p. 97

right cone ■ p. 97

sphere ■ p. 104

center ■ p. 104

radius ■ p. 104

Volume and Surface Area



The wheels shown in the photo above rotate on a ball bearing system. So, these wheels are often used on shopping carts, carts with shelves, television carts, or computer carts. In Lesson 2.8, you use ball bearings to explore the volume of a sphere.

2.1 Backyard Barbecue

Introduction to Volume and Surface Area ■ p. 59

2.2 Turn Up the Volume

Volume of a Prism ■ p. 65

2.3 Bending Light Beams

Surface Area of a Prism ■ p. 73

2.4 Modern Day Pyramids

Volume of a Pyramid ■ p. 79

2.5 Soundproofing

Surface Area of a Pyramid ■ p. 85

2.6 Making Concrete Stronger

Volume and Surface Area of a Cylinder ■ p. 91

2.7 Sand Piles

Volume and Surface Area of a Cone ■ p. 97

2.8 Ball Bearings and Motion

Volume and Surface Area of a Sphere ■ p. 103

2.1

Backyard Barbecue

Introduction to Volume and Surface Area

Objectives

In this lesson, you will:

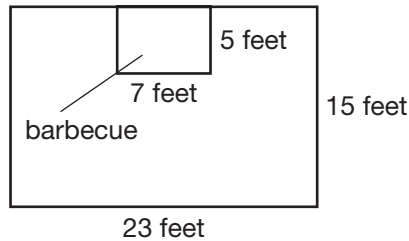
- Investigate the volume of a solid.
- Investigate the surface area of a solid.

Key Terms

- solid
- volume
- surface area



SCENARIO A landscaping company is putting in a stone patio and a brick barbecue on the patio in a client's backyard. A model of the patio with the barbecue is shown below.



2

Problem 1

Materials for the Patio



- What is the area of the entire patio? Show all your work and use a complete sentence in your answer.
- What is the area of the barbecue? Show all your work and use a complete sentence in your answer.
- The patio will be covered with stone tiles that are 12-inch squares. The tiles will not be put under the barbecue. What is the area that will be covered with the tiles? Show all your work and use a complete sentence in your answer.

How many tiles will be needed for the job? Show all your work and use a complete sentence in your answer.

Problem 1 Materials for the Patio



- D. When the company orders tiles, bricks, or stones, it always orders an extra 10% because some of the tiles could break during shipping or some of the tiles could break while they are being cut to fit the patio. How many tiles should the company order? Show all your work and use a complete sentence in your answer.



- E. Name the units that are used in the measurement of the length of the patio. Use a complete sentence in your answer.

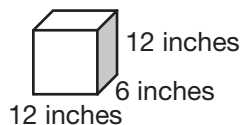
Name the units that are used in the measurement of the area of the patio. Use a complete sentence in your answer.

How are the units that are used to measure length different from the units that are used to measure area? Use a complete sentence in your answer.

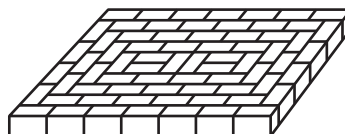
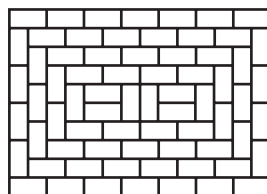
Investigate Problem 1



1. The barbecue will be made out of bricks that are 12 inches tall, 12 inches long, and 6 inches wide.



The base will be 4 feet tall. When the bricks are laid, they have to be staggered as much as possible so that the structure is solid. An overhead view and a side view of how the first layer of bricks might look are shown below.



Investigate Problem 1

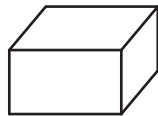
How tall is one layer of bricks in feet? Use a complete sentence in your answer.

How many layers will be needed to complete the base?
Use a complete sentence in your answer.

2. Use complete sentences to describe how you would use the length and the width of the barbecue base to find the number of bricks that are needed for one layer of the base.
3. Use complete sentences to describe how you would determine the number of bricks that are needed to complete the base of the barbecue.
4. How many measurements are involved in the planning of the construction of the barbecue? Describe these measurements. Use a complete sentence in your answer.



5. **Just the Math: Volume** The **volume** of a solid three-dimensional object, like the base of the barbecue, is the amount of space contained inside the object. A model of the base is shown below. Label the base with the measurements you identified in Question 4.



How many dimensions does volume involve?

Investigate Problem 1



6. Name the units that are used to indicate the volume of the base of the barbecue. Use a complete sentence in your answer.

7. Find the volume of the base of the barbecue. Use a complete sentence in your answer.



8. Complete the following statement. Then use complete sentences to explain your reasoning.

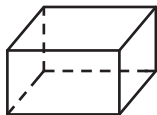
Area is to square feet as volume is to _____.



9. When you are considering the area of a two-dimensional plane figure like a polygon, the area is enclosed by one-dimensional line segments. When you are considering the volume of a three-dimensional solid figure like the barbecue base, what kind of figures are enclosing the volume? Use a complete sentence in your answer.

10. What is the shape of each side of the barbecue base? Use a complete sentence in your answer.

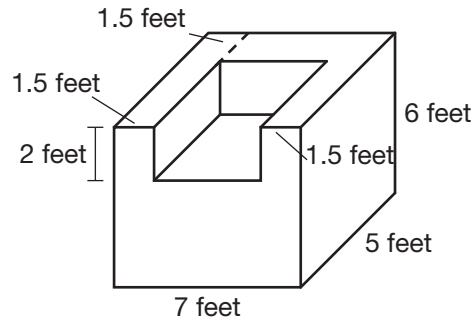
11. **Just the Math: Surface Area** Another important measurement of solids, such as the barbecue base, is the *surface area* of the solid. The **surface area** is the total area of the outside surfaces of the solid. How many outside surfaces does the barbecue base have? A model of the base is shown below. Describe these surfaces. Use complete sentences in your answer.



Investigate Problem 1



12. The entire barbecue is shown below. Find the volume of the barbecue. Show all your work and use a complete sentence in your answer.



2.2

Turn Up the Volume

Volume of a Prism

Objectives

In this lesson, you will:

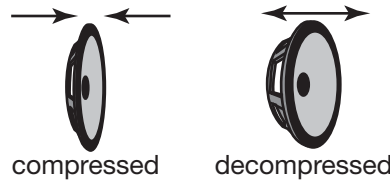
- Identify parts of prisms.
- Name prisms.
- Find volumes of prisms.

Key Terms

- polyhedron
- faces
- prism
- bases
- lateral faces
- height
- volume



SCENARIO Sound is produced by a speaker when a flexible cone inside the speaker vibrates back and forth. The cone vibrates by compressing and decompressing.

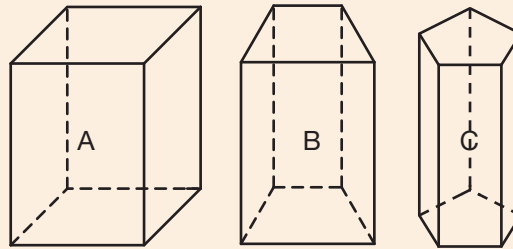


Because of this vibration, a speaker is usually enclosed in a rigid cabinet that can absorb these vibrations. Otherwise the sound coming out of the speaker would get drowned out by the sound of material being vibrated around the speaker, such as the floor upon which a speaker sits.

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Problem 1 Speaker Design

Speaker cabinets come in a variety of sizes and shapes. The size and shape depends on how the speaker is to be used. A few speaker cabinet designs are shown below. The dashed lines indicate the shapes of the sides of the cabinets that cannot be seen.



- A.** The speaker cabinets above are *polyhedrons*. A **polyhedron** is a solid that is formed from polygons. The polygons that form the solid are called **faces**. How are the three cabinets similar? How are the three cabinets different? Use complete sentences in your answer.

Problem 1 Speaker Design



- B.** Consider the polygons that form the tops of the speaker cabinets. What kind of polygon is each speaker cabinet top? Describe these polygons as precisely as possible. Use complete sentences in your answer.
- C.** Consider the polygons that form the sides of each speaker cabinet. What kind(s) of polygons are the sides of each speaker cabinet? Use a complete sentence in your answer.
- D.** Consider the polygons that form the bottoms of the speaker cabinets. What kind of polygon is each speaker cabinet bottom? Describe these polygons as precisely as possible. Use a complete sentence in your answer.
- E.** What do you notice about the top and bottom of each speaker cabinet? Use a complete sentence in your answer.

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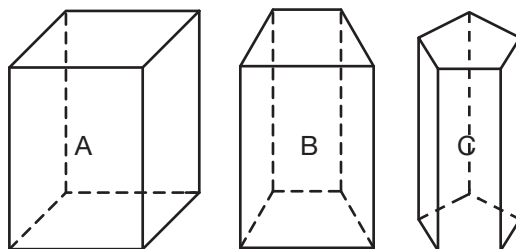
Investigate Problem 1



Take Note

Remember that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. So a rectangle is a special kind of parallelogram. A prism whose lateral faces are rectangles is called a **right prism** and a prism whose lateral faces are not rectangles is called an **oblique prism**. In this chapter, we will only be considering right prisms.

- 1. Just the Math: Prisms** The polyhedrons in Problem 1 are special polyhedrons called *prisms*. A **prism** is a polyhedron that has two congruent and parallel faces, called **bases**. The other faces of the prism are formed by joining the corresponding vertices of the bases. These faces are parallelograms and are called the **lateral faces** of the prism. The **height** of a prism is the length of a segment that is perpendicular to the bases.



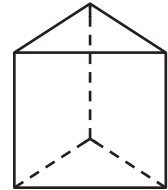
Investigate Problem 1

Identify the number of bases and lateral faces that each prism in Question 1 has. Use complete sentences in your answer.

How is the number of lateral faces related to the base? Use a complete sentence in your answer.



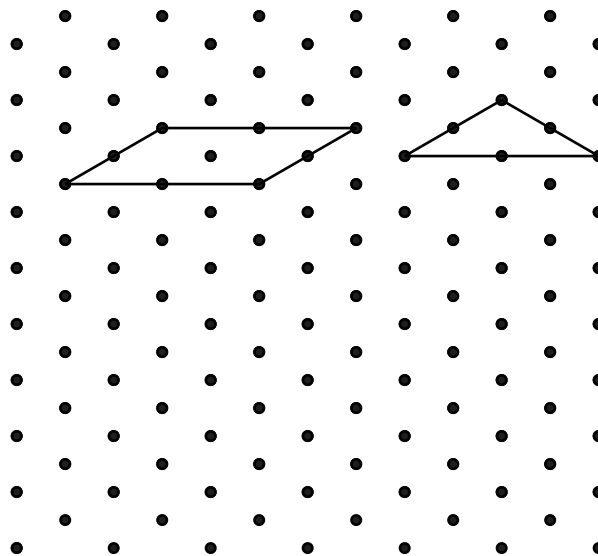
2. You can name prisms by the shape of their bases. For instance, the prism at the right is a triangular prism.



Identify the prisms in Question 1 by the shape of the bases.



3. You can use the dot grid below to draw your own prisms. The top bases of a rectangular prism and a triangular prism are already drawn for you. Complete each prism by drawing another base directly below the first base. You can go down the grid to your desired height. Then connect the corresponding vertices of the bases.



Take Note

The rectangular base of the prism looks like a parallelogram because of the viewpoint. If you drew the rectangle with right angles, then you wouldn't be able to see all of the faces of the prism.

Problem 2 The Space Inside the Speakers

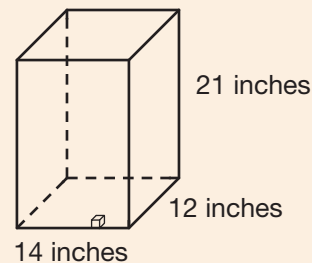


The amount of space inside the speaker cabinet is important because it can affect the sound coming out of a speaker. Consider the speaker cabinet below. This cabinet is in the shape of a rectangular prism.

A rectangular prism has three dimensions: length, width, and height.



- A.** Imagine a layer of cubes on the bottom of the speaker cabinet shown. Each cube is one inch long, one inch wide, and one inch tall. How many cubes are in the layer? Use a complete sentence in your answer.



Use a complete sentence to explain how you found your answer to part (A).

- B.** Suppose that you add another layer of cubes onto the existing layer. How many cubes are there in the two layers? Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (B).

- C.** Suppose that you add another layer of cubes onto the first two layers. How many cubes are there in the three layers? Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (C).

Take Note

If all of the faces of a right prism are congruent squares, then the prism is called a **cube**.

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Problem 2 The Space Inside the Speakers

- D. How many cubes will it take to fill the speaker cabinet?
Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (D).

2

Investigate Problem 2



1. In part (A), how does the number of cubes in one layer relate to the area of the base of the prism? Use a complete sentence in your answer.



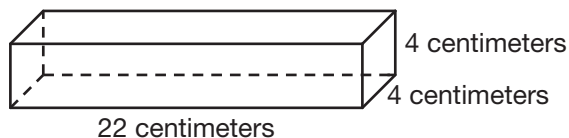
2. **Just the Math: Volume of a Prism** In Problem 2, you found the *volume* of, or the amount of space occupied by, the prism. You can find the area of any prism by multiplying the area of one of the bases by the height of the prism. This is written mathematically as

$$V = Bh$$

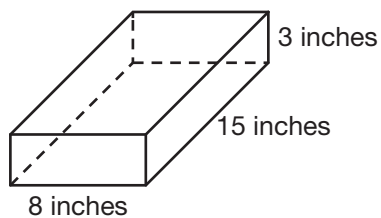
where V is the volume, B is the area of one of the prism's bases, and h is the prism's height. Volume is measured in cubic units because it is the product of three dimensions. What is the volume of the speaker cabinet in Problem 2? Use a complete sentence in your answer.

Write a formula for the volume of a rectangular prism with a length of ℓ units, a width of w units, and a height of h units.

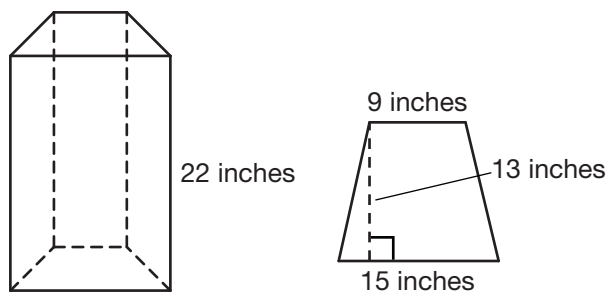
3. Find the volume of each rectangular prism. Show all your work and use a complete sentence in your answer.



Investigate Problem 2



4. The speaker cabinet shown below is designed so that the sound is pushed outward. The dimensions of one of the bases are also shown. Describe the shape of the speaker cabinet. Use a complete sentence in your answer.



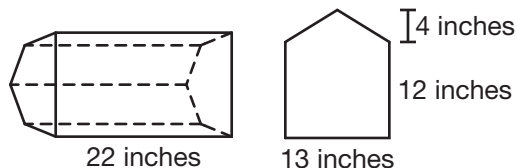
What is the area of a base of the speaker cabinet? Show all your work and use a complete sentence in your answer.

What is the height of the speaker cabinet? Use a complete sentence in your answer.

Find the volume of the speaker cabinet. Show all your work and use a complete sentence in your answer.

Investigate Problem 2

5. The speaker cabinet shown below is designed so that the sound is pushed upward. The dimensions of one of the bases are also shown. Describe the shape of the speaker cabinet. Use a complete sentence in your answer.



What is the area of a base of the speaker cabinet? Show all your work and use a complete sentence in your answer.

How did you find the area of the base? Use a complete sentence in your answer.

What is the height of the speaker cabinet? Use a complete sentence in your answer.

Find the volume of the speaker cabinet. Show all your work and use a complete sentence in your answer.



2.3

Bending Light Beams

Surface Area of a Prism

Objectives

In this lesson, you will:

- Find surface areas of prisms.
- Find volumes of prisms.
- Find the minimum surface area of a rectangular prism with a fixed volume.

Key Terms

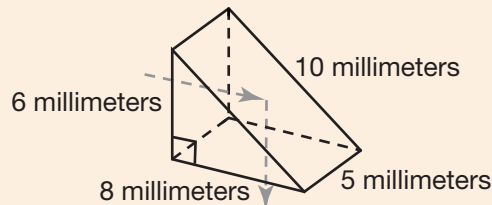
- lateral area
- surface area



SCENARIO Prisms are used in optical instruments like binoculars, telescopes, and microscopes. Prisms are used to refract (or bend) light beams, reflect light beams, or disperse (break up) light beams into separate colors. Optical prisms can be made of glass or other transparent materials that allow light to pass through them.

Problem 1 Following the Path of a Light Beam

Triangular prisms like the one below reflect light beams. The arrows show the path of a light beam as it travels through the prism.



The faces of optical prisms can be coated with a material so that they are not reflective and do not interfere with the path of the light beam.

- What is the height of the prism? Use a complete sentence in your answer.
- What is the area of a base of the prism? Show all your work and use a complete sentence in your answer.
- Write an expression for the total area of the bases. Then find this area. Use a complete sentence in your answer.
- Are the areas of the lateral faces all the same? Why or why not? Use complete sentences to explain your reasoning.



Problem 1 Following the Path of a Light Beam

- E. For each lateral face, identify the length and width of the face. Use complete sentences in your answer.
- F. Write an expression for the total area of the lateral faces. Then find this area. This area is called the **lateral area** of a prism. Use a complete sentence in your answer.
- G. What is the total area of the bases and the lateral faces? This area is called the *surface area* of the prism. Use a complete sentence in your answer.

Investigate Problem 1

1. Consider the lengths and widths of the lateral faces that you identified in part (E). What, if anything, do these dimensions have in common? Use a complete sentence in your answer.

Consider the expression that you wrote in part (F). Factor out the common dimension from the expression.

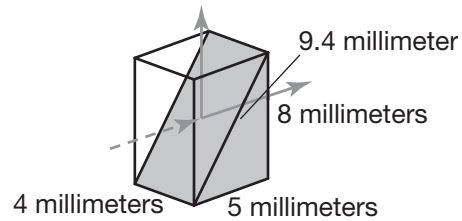
How is the sum that results from factoring related to the base of the prism? Use a complete sentence in your answer.

2. Write a formula for the surface area of a prism. Use S for the surface area, B for the area of the base, P for the perimeter of the base, and h for the height of the prism.

Do you think that your formula will work for prisms other than those with a triangular base? Use complete sentences to explain your reasoning.

Investigate Problem 1

3. A beamsplitter is a rectangular prism that allows part of a light beam to pass straight through the prism while bending the other part of the light beam as shown by the arrows below. A beamsplitter is formed from two triangular prisms.



Identify the dimensions of the bases and the lateral faces of the beamsplitter. Use complete sentences in your answer.

Use your formula from Question 2 to find the surface area of the rectangular prism. Show all your work and use a complete sentence in your answer.

Draw a diagram of each of the triangular prisms that form the beamsplitter. Label each diagram with its dimensions.

Consider the triangular prisms that form the beamsplitter. How do you think that the sum of the surface areas of these prisms will compare to the surface area of the beamsplitter? Use complete sentences to explain your reasoning.

Investigate Problem 1

Find the surface area of each triangular prism that forms the beamsplitter. Use your formula from Question 2. Show all your work and use a complete sentence in your answer.

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4. The table below shows the dimensions for different beamsplitters. Complete the table to show the volume and surface area of each beamsplitter.

Length (mm)	Width (mm)	Height (mm)	Volume (cu mm)	Surface area (sq mm)
4	4	18		
5	4	8		
5	2	8		
5	4	6		
5	7	10		
6	6	8		
7	4	8		
8	6	8		

5. Does a rectangular prism with a larger volume always have a larger surface area? Use complete sentences to explain your reasoning.
6. Does a rectangular prism with a larger surface area always have a larger volume? Use complete sentences to explain your reasoning.

Investigate Problem 1



7. Suppose that you have a rectangular prism with a volume of 18 cubic inches. The dimensions of the rectangular prism are whole inches. What are the dimensions of the rectangular prism with the smallest surface area and the given volume? Show all your work and use complete sentences to explain how you found your answer.

2.4

Modern Day Pyramids

Volume of a Pyramid

Objectives

In this lesson, you will:

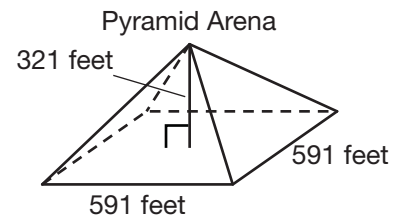
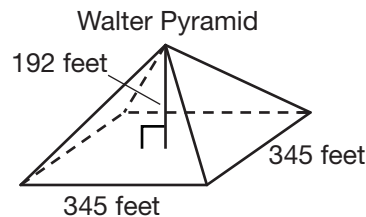
- Identify pyramids.
- Find volumes of pyramids.

Key Terms

- pyramid
- vertex
- lateral faces
- regular pyramid
- height
- slant height



SCENARIO In the United States, there are two *pyramids* that were built to be used as sports arenas. One of the pyramids is the Walter Pyramid, located in Long Beach, California. The other pyramid is the Pyramid Arena, located in Memphis, Tennessee. Scale models of these pyramids are shown below.



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Problem 1 Pyramid or Prism



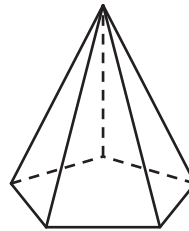
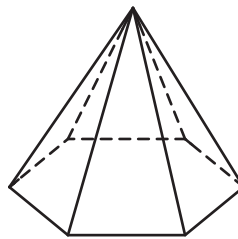
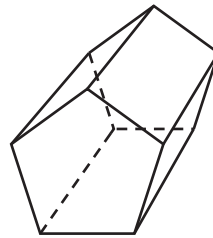
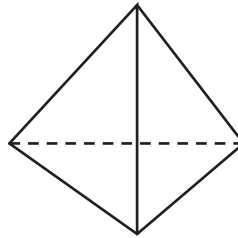
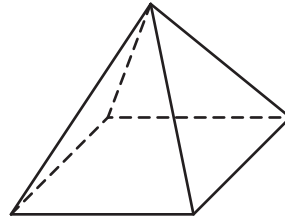
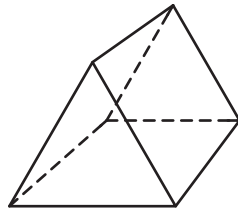
- A.** How are the pyramids the same? Use complete sentences in your answer.
- B.** How are the pyramids different? Use complete sentences in your answer.
- C.** How is a pyramid different from a prism? Use complete sentences in your answer.
- D.** Describe how you might form a pyramid by starting with a prism with the same base. Use complete sentences in your answer.

Investigate Problem 1



1. **Just the Math: Pyramids** A **pyramid** is a polyhedron that has a base that is a polygon and triangular faces that meet at one point. This point is called the **vertex** of the pyramid. The triangular faces are called the **lateral faces** of the pyramid. A pyramid, like a prism, is named for the shape of its base. What kinds of pyramids are the Pyramid Arena and the Walter Pyramid? Use a complete sentence in your answer.

2. Identify each polyhedron below as completely as possible.

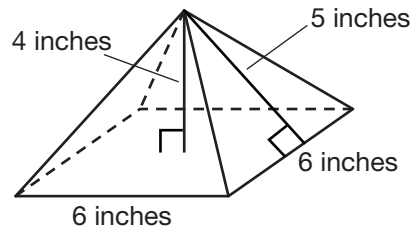


3. How does the number of lateral faces of a pyramid relate to the number of sides of the base? Use a complete sentence in your answer.

Investigate Problem 1

4. Just the Math: Measurement in a Regular Pyramid

A **regular pyramid** is a pyramid in which the base is a regular polygon and the *height* meets the base at its center. We will only consider regular pyramids in this chapter. The **height** of a regular pyramid is the perpendicular distance from the vertex to the base. The **slant height** of a regular pyramid is the altitude of the lateral faces. Identify the measurements that represent the height and slant height on the regular pyramid below.



Is the slant height for all of the lateral faces the same?
How do you know? Use complete sentences in your answer.

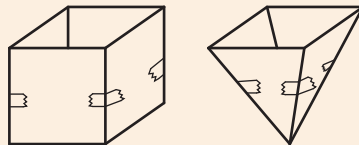
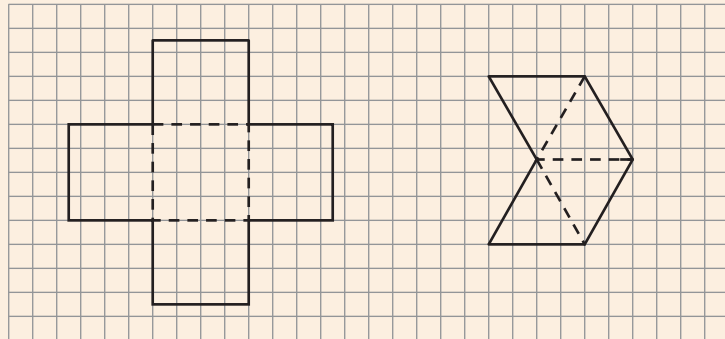
Take Note

Remember that a regular polygon is a polygon in which the side lengths are congruent and the angle measures are congruent.

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Problem 2 Pyramids and Volume

- A. Draw the figures below on a sheet of graph paper exactly as shown. Then cut out the figures and fold along the dashed lines to form an open prism and an open pyramid. Tape the sides together as shown so that the sides do not overlap.



Problem 2 Pyramids and Volume

- B.** How do the side lengths of the bases and the heights of your models compare? Use complete sentences in your answer.
- C.** Which model do you think has a greater volume? Use complete sentences to explain your reasoning.
- D.** Fill the pyramid with rice or dried beans. Then dump the rice or beans from the pyramid into the prism. Repeat this process until the prism is full. How many times did you fill the pyramid? Use a complete sentence in your answer.
- E.** How do you think that the volume of a pyramid relates to the volume of a prism when the bases are identical and the heights are the same? Use a complete sentence in your answer.

Investigate Problem 2



1. Write a formula for the volume of a pyramid. Use V for the volume, B for the area of the base, and h for the height.
2. Find the volumes of the Pyramid Arena and the Walter Pyramid. Show all your work and use a complete sentence in your answer.



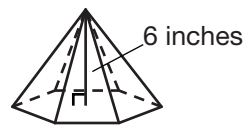
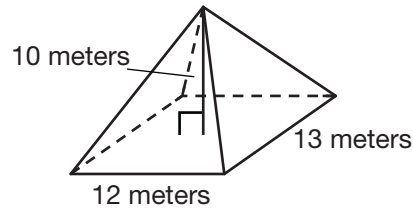
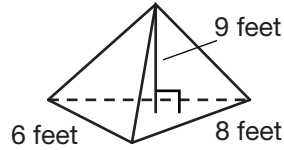
3. Another famous modern-day pyramid sits in front of the Louvre art museum in Paris, France. The pyramid was designed by I. M. Pei, a well known Chinese-American architect. This square pyramid has a base that has a side length of 115 feet and a height of about 70 feet. Draw a model of this pyramid.

Investigate Problem 2

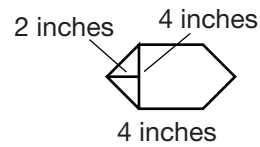
Find the volume of this pyramid. Show all your work and use a complete sentence in your answer.



4. Identify the pyramid. Then find its volume. Show all your work and use a complete sentence in your answer.



Base of pyramid:



2.5

Soundproofing

Surface Area of a Pyramid

Objectives

In this lesson, you will:

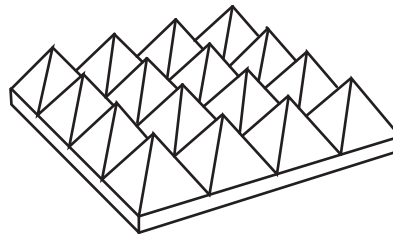
- Find surface areas of pyramids.
- Find surface areas of composite solids.

Key Terms

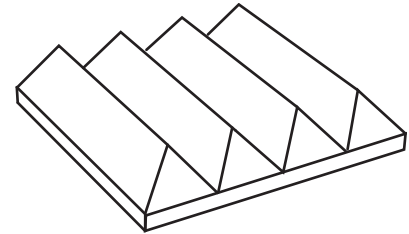
- lateral area
- composite solid



SCENARIO In small recording studios and large auditoriums, acoustical foam is placed on walls and ceilings to control sound and correct sound problems. The size and shape of the foam that is used depends on the sound problem. Two different acoustical foam shapes are shown below.



Pyramidal foam



Wedge foam

2

Problem 1

Area of a Foam Surface

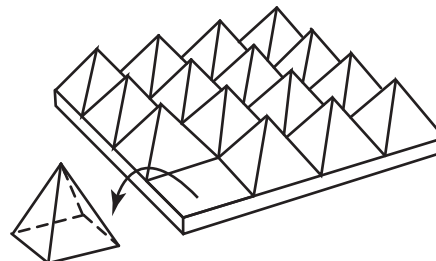


- A. Describe the shapes that form each kind of foam. Use complete sentences in your answer.
- B. Which foam do you think has the greater area on its top surface? Use complete sentences to explain your reasoning.

Investigate Problem 1



1. The pyramidal foam is formed from a rectangular prism on the bottom and many square pyramids placed on top of the rectangular prism as shown.

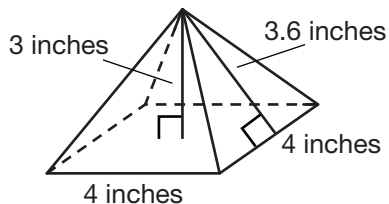


Investigate Problem 1

Are the pyramids regular pyramids? Use a complete sentence to explain your reasoning.



2. Consider one of the pyramids from the foam. Identify the dimensions of the base and the lateral faces. Use complete sentences in your answer.



3. How would you find the surface area of the pyramid from Question 2? Use complete sentences in your answer.

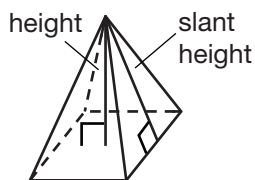
To find the area of one of the lateral faces, what dimensions of the pyramid will you need to use? Use a complete sentence in your answer.

4. Write an expression for the area of the base of the pyramid. Then find the area of the base. Use a complete sentence in your answer.

5. Write an expression for the *lateral area* of the pyramid. The **lateral area** of a pyramid is the sum of the areas of the lateral faces. Then find the lateral area. Use a complete sentence in your answer.

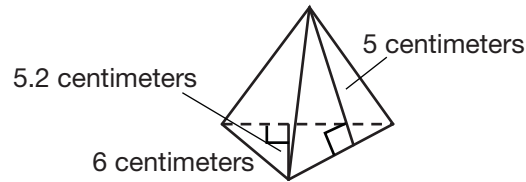
Take Note

Remember that the height of a pyramid is the perpendicular distance from the vertex to the center of the base and the slant height is the altitude of the lateral faces.

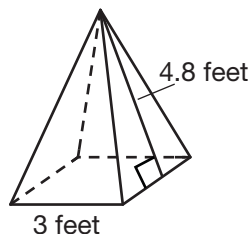


Investigate Problem 1

6. What is the surface area of the pyramid in Question 2? Show all your work and use a complete sentence in your answer.
7. Consider the lateral area from Question 5. What dimensions from the base of the pyramid are used to find the lateral area? Use a complete sentence in your answer.
8. Write a formula for the surface area of a regular pyramid. Use S for the surface area, B for the area of the base, P for the perimeter of the base, and ℓ for the slant height of the pyramid.
9. Use the formula from Question 8 to find the surface area of each regular pyramid. Show all your work and use a complete sentence in your answer.



Investigate Problem 1



2

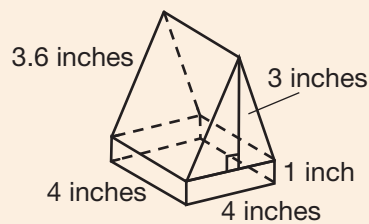
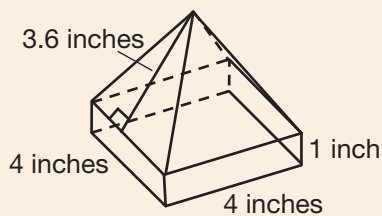
Problem 2 Composite Solids



Each piece of acoustical foam shown in Problem 1 is a *composite solid*. A **composite solid** is formed from two or more solids.



- A. Consider the small pieces of foam below. Describe the solids that form each piece of foam. Use complete sentences in your answer.



- B. How many faces does each composite solid from part (A) have? Use a complete sentence in your answer.
- C. Which solid do you think has a greater surface area? Use a complete sentence to explain your reasoning.

Problem 2 Composite Solids

D. Find the surface area of each composite solid. Show all your work and use a complete sentence in your answer.



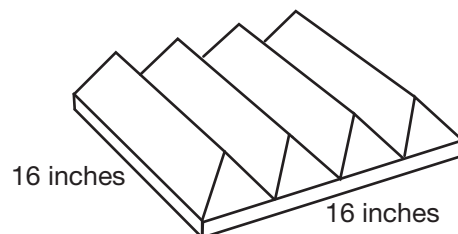
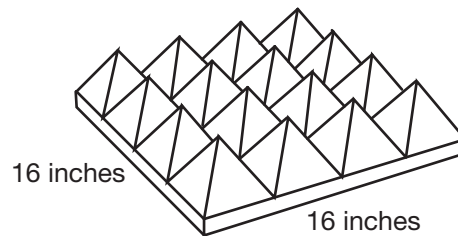
E. Were you surprised by your answer to part (D)? Use complete sentences to explain your reasoning.

2

Investigate Problem 2



1. Consider the larger pieces of the acoustical foam shown below. The pieces of foam are formed from the small pieces in part (A). How many faces does each solid have? Use a complete sentence in your answer.



Investigate Problem 2

2. If you consider only the faces of the foam that are above the 1-inch bottom prism, how many faces does each solid have? Use a complete sentence in your answer.

Which collection of faces do you think has the greater surface area? Use complete sentences to explain your reasoning.

3. Find the surface area of each collection of faces described in Question 2. Show all your work and use a complete sentence in your answer.

4. Which collection has the greater surface area? Use a complete sentence in your answer.

Does this answer surprise you? Explain your reasoning. Use complete sentences in your answer.

How do you think that these soft, shaped surfaces control sound? Use complete sentences in your answer.



2.6

Making Concrete Stronger

Volume and Surface Area of a Cylinder

Objectives

In this lesson, you will:

- Find volumes of cylinders.
- Find surface areas of cylinders.

Key Terms

- cylinder
- height



SCENARIO If you have ever seen highway construction workers preparing a roadway for a concrete surface, you might have seen the workers laying out steel reinforcing bars, or rebar for short. The rebar makes the concrete surface stronger.

Problem 1 Reinforcing Concrete

A simple model of a piece of rebar is shown below. Rebar has small raised grooves on it so that it is better integrated with the concrete. The model below doesn't include the grooves.



- A.** A view of one end of the rebar is shown below. What is the area of the end? Use 3.14 for π and round your answer to the nearest square millimeter if necessary. Use a complete sentence in your answer.



- B.** Describe the shape of the piece of rebar. Use a complete sentence in your answer.
- C.** How is this solid different from the solids that you have been investigating so far in this chapter? Use complete sentences in your answer.



2

Investigate Problem 1

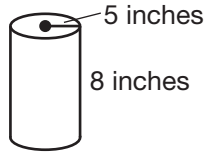


Take Note

A **right cylinder** is a cylinder in which the segment that connects the centers of the bases is perpendicular to the bases. We will only consider right cylinders in this chapter.

2

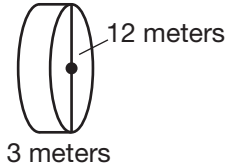
1. **Just the Math: Cylinder** A piece of rebar is in the shape of a *cylinder*. A **cylinder** is a solid that has two congruent parallel bases that are circles. The **height** of a cylinder is the perpendicular distance between bases. Identify the radius, diameter, and height of each cylinder.



Radius: _____

Diameter: _____

Height: _____



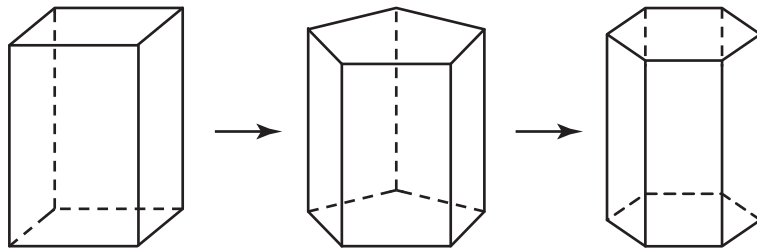
Radius: _____

Diameter: _____

Height: _____



2. Consider the square prism shown below at the left. Then imagine that the bases change to regular pentagons as shown in the center. Now imagine that the bases change to regular hexagons as shown on the right.



Suppose that you continue this process and keep increasing the number of sides in the polygons that form the bases. What shape do you think the bases will eventually form? Use complete sentences to explain your reasoning.

How does this help you figure out how to find the volume of a cylinder? Use a complete sentence in your answer.

Investigate Problem 1



- 3. Just the Math: Volume of a Cylinder** The volume V of a cylinder with radius r and height h is given by

$$V = \pi r^2 h.$$

Use complete sentences to explain in your own words how to find the volume of a cylinder in terms of the area of a base.



- 4.** A piece of rebar is 75 centimeters long and has a diameter of 13 millimeters. What is the diameter of the piece of rebar in centimeters? Show all your work and use a complete sentence in your answer.

Take Note

Whenever you are making a calculation that involves one or more measurements, the measurements should all use the same units. For instance, if a calculation involves several measurements of time, all of the times should be in hours, or all of the times should be in seconds, and so on.

What is the total volume of this piece of rebar? Show all your work and use a complete sentence in your answer. Use 3.14 for π and round your answer to the nearest tenth if necessary.

- 5.** Suppose that a construction worker cuts a section of the rebar that is described in Question 4. This section of rebar is one centimeter long and weighs approximately 0.01 kilogram. Find the weight of the piece of rebar from Question 4. Show all your work and use a complete sentence in your answer.

Investigate Problem 1



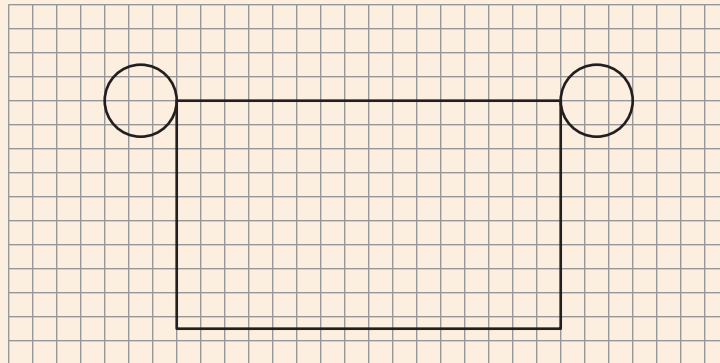
6. Rebar is also used in concrete sidewalks. A sidewalk is being made. A worker needs to use 3 pieces of rebar that are each 300 centimeters long and 10 pieces of rebar that are each 90 centimeters long. The diameter and weight of each piece of rebar are the same as in Questions 4 and 5. How much steel in cubic centimeters will be in the sidewalk? How much will it weigh? Use 3.14 for π . Show all your work and use a complete sentence in your answer.

2

Problem 2 Surface of a Cylinder



- A. Draw the figure below on a sheet of graph paper exactly as shown. Then cut out the figure and tape the longer sides together to form a cylinder.



- B. What shapes were used to form your model of a cylinder? Use a complete sentence in your answer.

Problem 2 Surface of a Cylinder

- C. Use your model to help you explain how you would find the surface area of a cylinder. Use complete sentences in your answer.

Investigate Problem 2



1. Consider the rectangle on the grid in part (A). What is the length of this rectangle? Use a complete sentence in your answer.

How does the width of the rectangle relate to the circle?
Use a complete sentence in your answer.

Find the width of the rectangle. Show all your work and use a complete sentence in your answer. Use 3.14 for π .

Find the area of the rectangle. Use a complete sentence in your answer.

2. Find the area of the bases of the cylinder in Problem 2. Show all your work and use a complete sentence in your answer. Use 3.14 for π .

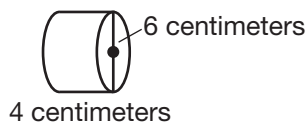
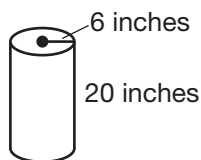
3. Find the surface area of the cylinder in Problem 2. Show all your work and use a complete sentence in your answer.

Investigate Problem 2

4. Write a formula for the surface area of a cylinder. Use S for the surface area, r for the radius, and h for the height.



5. Find the surface area of each cylinder. Show all your work and use a complete sentence in your answer. Use 3.14 for π .



6. Find the surface area of a piece of rebar that has a diameter of 1.3 centimeters and a height of 75 centimeters. Show all your work and use a complete sentence in your answer. Use 3.14 for π and round your answer to the nearest tenth.



2.7

Sand Piles

Volume and Surface Area of a Cone

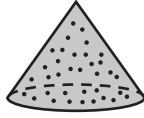
Objectives

In this lesson, you will:

- Find volumes of cones.
- Find surface areas of cones.



SCENARIO When sand, dirt, or other like materials are freely dumped onto a horizontal surface, the material forms into a pile that is shaped like a *cone*.



Key Terms

- cone
- height
- right cone
- slant height
- lateral area



Problem 1

Adding to the Pile

- Describe the shape of a cone using the figure of the sand pile above. Use a complete sentence in your answer.
- Is this solid a polyhedron? Why or why not? Use a complete sentence in your answer.
- Describe the shape of the base of the sand pile. Use a complete sentence in your answer.
- Name other objects that you know are shaped like a cone.

Investigate Problem 1

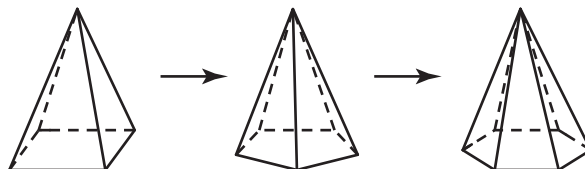
- Just the Math: Cone** A **cone** is a solid that has one circular base and one vertex that is not in the same plane as the base. The **height** of a cone is the perpendicular distance from the vertex to the base. When the height meets the base at its center, the cone is a **right cone**. In this chapter, we will only consider right cones. The **slant height** of a right cone is the distance from the vertex to a point on the edge of the base. How is a cone similar to a pyramid? How is a cone different from a pyramid? Use complete sentences in your answer.

Investigate Problem 1

2. How is a cone similar to a cylinder? How is a cone different from a cylinder? Use complete sentences in your answer.



3. Consider the square pyramid shown below on the left. Imagine that the base changes to a regular pentagon as shown in the center. Now imagine that the base changes to a regular hexagon as shown on the right.



Suppose that you continue this process and keep increasing the number of sides in the polygon that forms the base. What shape do you think the base will eventually form? Use a complete sentence to explain your reasoning.

How does this help you figure out how to find the volume of a cone? Use a complete sentence in your answer.



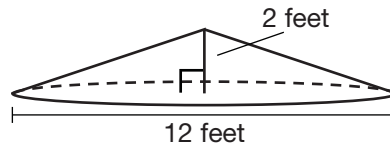
4. **Just the Math: Volume of a Cone** The volume V of a cone with radius r and height h is given by

$$V = \frac{1}{3}\pi r^2 h.$$

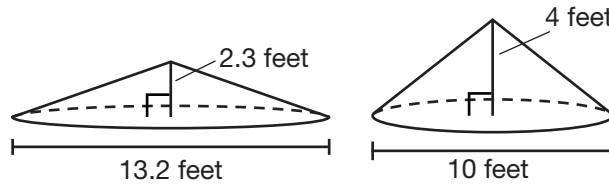
Use complete sentences to explain in your own words how to find the volume of a cone in terms of the area of the base.

Investigate Problem 1

5. Find the volume of the sand pile below. Show all your work and use a complete sentence in your answer. Use 3.14 for π .



6. The height and radius of the pile not only depend on the amount of material that is dumped but also depend on the kind of material that is dumped. The pile on the left is a pile of sand, and the pile on the right is a pile of stones. Find the volume of each pile. Show all your work and use a complete sentence in your answer. Use 3.14 for π . Round your answer to the nearest cubic foot.



How do the volumes compare? Use a complete sentence in your answer.

Why do you think the piles have different heights and diameters? Use complete sentences to explain your reasoning.

Investigate Problem 1

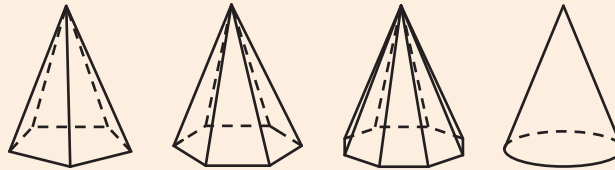
Do you think that the volumes of these piles are entirely made up of sand and stone? Why or why not? Use complete sentences in your answer.

Problem 2 Covering a Pile

2



- A. Consider the solids below. Describe how these solids are alike and how they are different. Use complete sentences in your answer.



- B. Find the surface area of the regular pentagonal pyramid above. The side length of the base is 2.4 feet; the area of the base is 9.6 square feet; and the slant height is 10.1 feet. Show all your work and use a complete sentence in your answer.
- C. Find the surface area of the regular hexagonal pyramid above. The side length of the base is 2 feet; the area of the base is 10.2 square feet; and the slant height is 10.1 feet. Show all your work and use a complete sentence in your answer.

Problem 2 Covering a Pile

- D.** Find the surface area of the regular octagonal pyramid on the previous page. The side length of the base is 1.5 feet; the area of the base is 10.8 square feet; and the slant height is 10.2 feet. Show all your work and use a complete sentence in your answer.
- E.** How do you think you can find the surface area of the cone? Use complete sentences to explain your reasoning.
- F.** Find the surface area of the cone. The radius of the base is 2 feet, and the slant height is 10.2 feet. Show all your work and use a complete sentence in your answer. Use 3.14 for π . Round your answer to the nearest tenth.

2

Investigate Problem 2



- 1. Just the Math: Surface Area of a Cone** The surface area S of a cone with radius r and slant height ℓ is given by

$$S = \pi r^2 + \pi r \ell.$$



The lateral area of a cone is the part of the cone that does not include the base. What part of the formula for the surface area of a cone represents the lateral area of the cone?

Investigate Problem 2

Use complete sentences to explain how you can determine the formula for the lateral area of a cone by using the formula for the lateral area of a pyramid.

2



2. Consider the piles of material in Question 6 of Problem 1. Find the surface area of the cones that represent each pile. Show all your work and use a complete sentence in your answer. Use 3.14 for π . Round your answers to the nearest tenth.



How do the volumes of these cones compare? How do the surface areas compare? Do the answers surprise you? Why or why not? Use complete sentences in your answer.



Objectives

In this lesson, you will:

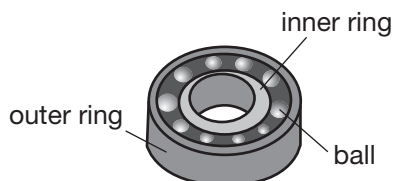
- Find volumes of spheres.
- Find surface areas of spheres.

Key Terms

- sphere
- center
- radius



SCENARIO If you have ever used inline skates or a skateboard, you have benefited from the smooth motion created by ball bearings. A ball bearing is shown below. The three basic parts of a ball bearing are an inner ring, an outer ring, and a number of perfect balls. The balls between the rings allow the outer ring to spin smoothly while the inner ring remains stationary.

**Problem 1****Moving Out into Space**

- A. If possible, name the geometric shape of the balls in a ball bearing. Use a complete sentence in your answer.
- B. Is this shape a polyhedron? Why or why not? Use a complete sentence in your answer.
- C. Consider point A below. Draw eight different points below that are 3 centimeters from point A .

• A

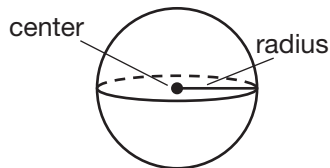
What geometric shape can you form from the points around point A ? Use a complete sentence in your answer.

- D. Now imagine that you can draw points that are 3 centimeters from point A and are in the space that is above and below this sheet of paper. If you could draw these points, what geometric shape would be formed from all your points around point A ? Use a complete sentence in your answer.

Investigate Problem 1



- 1. Just the Math: Sphere** In part (D), your points would form a *sphere*. A **sphere** is the set of all points in space that are the same distance from a given point. This given point is called the **center** of the sphere. The **radius** of a sphere is the distance from the center to any point on the sphere.



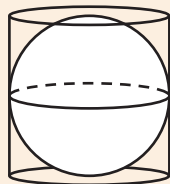
What are the radius and diameter of the sphere described in Problem 1? Use a complete sentence in your answer.

- 2.** How are the centers of spheres and circles similar? How are they different? Use complete sentences in your answer.
- 3.** How are the radii of spheres and circles similar? How are they different? Use complete sentences in your answer.

Problem 2 Filling a Sphere

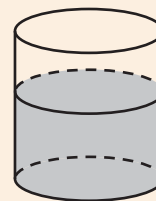


- A.** Use clay or modeling dough to make a sphere that has a diameter of three inches. Then wrap a strip of plastic around the sphere. Trim the plastic so that the strip of plastic is the same height as your sphere. Finally, tape the plastic together tightly to form a cylinder with an open top and open bottom as shown.



Problem 2 Filling a Sphere

- B.** Flatten the sphere so that it fits tightly into the bottom of the cylinder as shown.



Record the following dimensions.

Height of cylinder: _____

Height of flattened sphere: _____

Height of empty space in cylinder: _____

What relationship do you notice? Use a complete sentence in your answer.

- C.** Find the volumes of the cylinder and the flattened sphere. Show all your work and use a complete sentence in your answer. Leave your answers in terms of π .
- D.** How does the volume of the sphere relate to the volume of the cylinder? Use a complete sentence in your answer.

2

Investigate Problem 2



- 1. Just the Math: Volume of a Sphere** The volume V of a sphere with radius r is given by

$$V = \frac{4}{3}\pi r^3.$$



You should have seen in Problem 2 that the volume of a sphere is two thirds of the volume of a cylinder when the sphere and cylinder have the same height and radius. Show how the formula for the volume of a cylinder, $V = \pi r^2 h$ where r is the radius and h is the height, can be used to find the formula for the volume of a sphere.

Investigate Problem 2

2. A ball bearing contains eight balls. Each ball has a radius of three millimeters. Find the volume of one ball. Then find the volume of the balls in one bearing. Show all your work and use a complete sentence in your answer. Leave your answers in terms of π .

2



3. Suppose that you double the radius of the ball in Question 2. Do you expect the volume to double? Why or why not? Use a complete sentence in your answer.

Find the volume of this ball. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π .

How does the volume of this ball compare to the volume of one of the balls in Question 2? Use a complete sentence in your answer.

4. A “608 bearing” is the bearing that is used in most inline skates. One 608 bearing contains 7 steel balls, each with a diameter of approximately 4 millimeters. An inline skate has four wheels and each wheel uses a bearing. How much steel is in the balls in the ball bearings of a pair of inline skates? Show all your work and use a complete sentence in your answer. Use 3.14 for π and round your answer to the nearest cubic millimeter.



Problem 3 Covering a Sphere



The balls for ball bearings are made from a piece of steel wire. The balls are formed when a piece of steel wire is smashed from each end towards the middle. The result is a sphere with a ring around the middle, as shown. After the excess metal is broken and ground off, the entire surface of the ball is polished to get a smooth shiny surface. To discover how to find the area of the surface of a sphere, complete the steps below.



- A.** Choose a tennis ball, or a ball that is similar in size. Measure and record the diameter of the ball. Use a complete sentence in your answer.
- B.** Use the diameter from part (A) to draw five identical circles on paper with this diameter. Then cut out each of these circles.
- C.** Tape your circles to the surface of your ball so that the paper does not overlap. This can be difficult, so you can cut the circles to fill in the gaps. Use all of a circle before you start using another circle. How many circles did you use to completely cover the ball with no overlap? Use a complete sentence in your answer.

2

Investigate Problem 3

- 1.** If done perfectly, it should take exactly four circles to completely cover the ball in Problem 3. How does the surface area of a sphere relate to the area of a circle with the same radius? Use a complete sentence in your answer.

Write a formula for the surface area S of a sphere with radius r .

- 2.** Find the surface area of a ball from a 608 bearing. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π .
- 3.** Suppose that you double the radius of the ball in Question 2. Do you expect the surface area to double? Why or why not? Use a complete sentence in your answer.



Investigate Problem 3

Find the surface area of the ball bearing. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π .

How does the surface area of this ball compare to the surface area of the ball in Question 2? Use a complete sentence in your answer.

Take Note

Remember that V represents volume, S represents surface area, B represents the area of a base, P represents the perimeter of a base, ℓ represents slant height, h represents height, and r represents a radius.



Summary

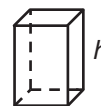
Volume and Surface Area Formulas

In this chapter, you came up with the volume and surface area formulas for common geometric solids:

- Volume and Surface Area of a Prism:

$$V = Bh$$

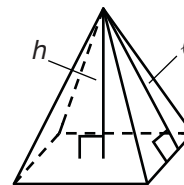
$$S = 2B + Ph$$



- Volume and Surface Area of a Pyramid:

$$V = \frac{1}{3}Bh$$

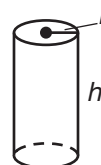
$$S = B + \frac{1}{2}P\ell$$



- Volume and Surface Area of a Cylinder:

$$V = \pi r^2 h$$

$$S = 2\pi r^2 + 2\pi rh$$



- Volume and Surface Area of a Cone:

$$V = \frac{1}{3}\pi r^2 h$$

$$S = \pi r^2 + \pi r\ell$$



- Volume and Surface Area of a Sphere:

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

