

# Looking Ahead to Chapter 1

1

## Focus

In Chapter 1, you will classify and describe polygons. You will find perimeter and area of polygons. You will also find circumference and area of circles.

## Chapter Warmup

Answer these questions to help you review skills that you will need in Chapter 1.

Evaluate the expression when  $x = 4$ .

1.  $x + 5$

2.  $17 - x$

3.  $10 + 3x$

4.  $14 - 5x$

5.  $\frac{5x}{2}$

6.  $\frac{3}{4}x - 7$

Complete the statement.

7. 5 feet = \_\_\_\_\_ inches

8. 15 centimeters = \_\_\_\_\_ meters

9. 3 yards = \_\_\_\_\_ feet

Read the problem scenario below.

You are designing a table top for a coffee table. The top of the table will be a 4-foot by 2-foot rectangle. You want to cover the table top with glass tiles. Each tile is a 2-inch by 2-inch square.

10. How many tiles will you need to cover the top of the table?

11. You have \$60. A box of 40 glass tiles costs \$8. Do you have enough money to buy the number of boxes that you need to cover the top of the table with tiles?

## Key Terms

polygon ■ p. 6

side ■ p. 6

triangle ■ p. 6

quadrilateral ■ p. 6

pentagon ■ p. 6

octagon ■ p. 7

perimeter of a polygon ■ p. 8

area of a polygon ■ p. 10

rectangle ■ p. 13

length ■ p. 15

width ■ p. 15

parallelogram ■ p. 17

base of a parallelogram ■ p. 19

height of a parallelogram

■ p. 19

vertex ■ p. 21

height of a triangle ■ p. 24

altitude ■ p. 24

base of a triangle ■ p. 24

trapezoid ■ p. 29

base of a trapezoid ■ p. 29

height of a trapezoid ■ p. 29

regular polygon ■ p. 34

apothem ■ p. 35

diameter ■ p. 39

circumference ■ p. 39

radius ■ p. 41

irrational number ■ p. 41

area of a circle ■ p. 45

composite figure ■ p. 49

# Perimeter and Area



Lighting is essential when taking photographs. The amount of light let in through the lens of a camera is controlled by a hole called the aperture. In Lesson 1.6, you will find the circumferences and areas of different-sized apertures.

## 1.1 Building a Deck

Introduction to Polygons, Perimeter, and Area ■ p. 5

## 1.2 Weaving a Rug

Area and Perimeter of a Rectangle and Area of a Parallelogram ■ p. 13

## 1.3 Sailboat Racing

Area of a Triangle ■ p. 21

## 1.4 The Keystone Effect

Area of a Trapezoid ■ p. 27

## 1.5 Traffic Signs

Area of a Regular Polygon ■ p. 33

## 1.6 Photography

Circumference and Area of a Circle ■ p. 39

## 1.7 Installing Carpeting and Tile

Composite Figures ■ p. 49

# Mathematical Representations

1

**INTRODUCTION** Mathematics is a human invention, developed as people encountered problems that they could not solve. For instance, when people first began to accumulate possessions, they needed to answer questions such as: How many? How many more? How many less?

People responded by developing the concepts of numbers and counting. Mathematics made a huge leap when people began using symbols to represent numbers. The first “numerals” were probably tally marks used to count weapons, livestock, or food.

As society grew more complex, people needed to answer questions such as: Who has more? How much does each person get? If there are 5 members in my family, 6 in your family, and 10 in another family, how can each person receive the same amount?

During this course, we will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand our world. The following processes can help you solve problems.



## Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do you understand it?
- What is the question that you are being asked? Does it make sense?



## Think for Yourself

- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?



## Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Do our reasoning and our answer make sense to one another?



## Work with Your Group

- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Do our reasoning and our answer make sense to one another?
- How can we explain our solution to one another? To the class?



## Share with the Class

- Here is our solution and how we solved it.
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?

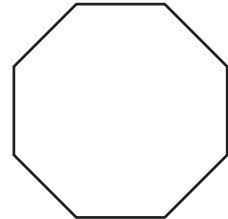
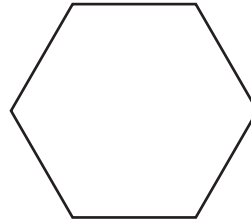
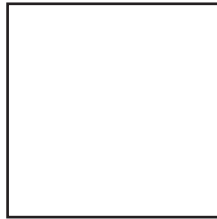
## Objectives

In this lesson, you will:

- Classify polygons by the number of sides.
- Find perimeters of polygons.
- Find areas of polygons.



**SCENARIO** Two homeowners have decided to put a deck in their backyard. They are considering the different deck shapes shown below.



## Key Terms

- polygon
- side
- triangle
- quadrilateral
- pentagon
- octagon
- perimeter
- area



### Problem 1

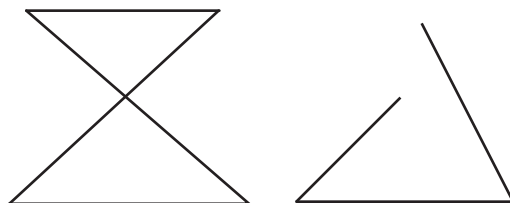
#### Choosing the Deck Shape

- A. How are the deck shapes similar to each other?  
Use complete sentences in your answer.
  
- B. How are the deck shapes different from each other?  
Use complete sentences in your answer.
  
- C. Describe the advantages one deck shape may have over the other deck shapes. Use complete sentences to explain your reasoning.
  
- D. Which deck shape would you use? Use complete sentences to explain your reasoning.

## Investigate Problem 1



- 1. Just the Math: Regular Polygons** Each deck shape is a *polygon*. A **polygon** is a plane figure that is formed from three or more line segments, called **sides**. Each side must meet exactly two other sides and the sides can have no points in common except for the *endpoints*. For instance, the figures below are *not* polygons.



Why do the figures above fail to be polygons? Use complete sentences in your answer.

## Take Note

Remember that all line segments have two **endpoints**. For instance, line segment  $AB$  below has points  $A$  and  $B$  as its endpoints.



- 2. Just the Math: Classifying Polygons** A polygon can be classified by its number of sides. Draw a polygon with three sides. What kind of polygon did you draw? Use a complete sentence in your answer.

Draw a polygon with four sides. This polygon is a **quadrilateral**.

Draw a polygon with five sides. This polygon is a **pentagon**.

## Take Note

A six-sided polygon is called a **hexagon**; a seven-sided polygon is called a **heptagon**; a nine-sided polygon is called a **nonagon**; a ten-sided polygon is called a **decagon**; a twelve-sided polygon is called a **dodecagon**. A polygon with  $n$  sides is called an  **$n$ -gon**.

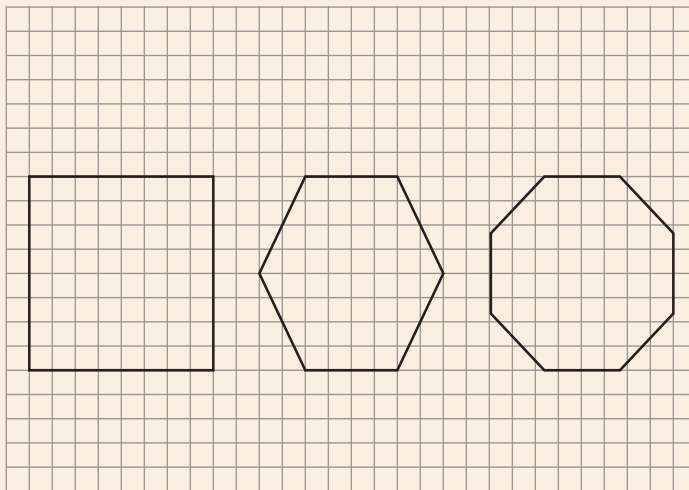
## Investigate Problem 1

Draw a polygon with eight sides. This polygon is an **octagon**.

- Classify each deck shape in Problem 1 by its number of sides. Use complete sentences in your answer.

## Problem 2 Planning a Deck

The homeowners have made scale *drawings* of their possible deck choices on graph paper as shown below. Each square represents a square that is one foot long and one foot wide.



### Take Note

A *scale drawing* of an object is a drawing that is a miniature version of the actual object. The purpose of a scale drawing is to give you a realistic idea of how an object will look. You will learn more about scale drawings in Chapter 7.



- The homeowners will enclose their entire deck with a railing. If they choose the quadrilateral shape, how many feet of railing will they need? Explain how you found the result. Use complete sentences in your answer.
- If they choose the hexagon shape, approximately how many feet of railing will they need? Explain how you found the result. Use complete sentences in your answer.

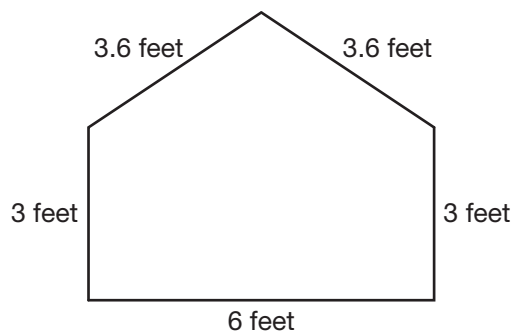
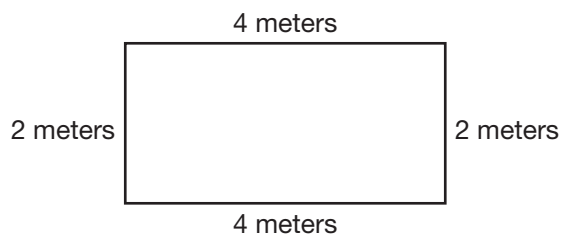
## Problem 2 Planning a Deck

- C. If they choose the octagon shape, approximately how many feet of railing will they need? Explain how you found the result. Use complete sentences in your answer.
- D. Which plan would you choose based on your answers to parts (A) through (C) or any other factors that you think would be important? Use complete sentences to explain your reasoning.

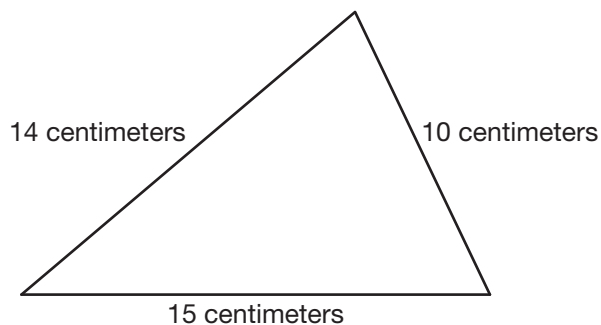
## Investigate Problem 2



1. **Just the Math: Perimeter** In Problem 2, you found the **perimeter**, or the distance around, each polygon. Use a complete sentence to explain how to find the perimeter of any polygon.
2. Find the perimeter of each polygon below. Show all your work and use a complete sentence in your answer. Be sure that you include the units in your answer.



## Investigate Problem 2



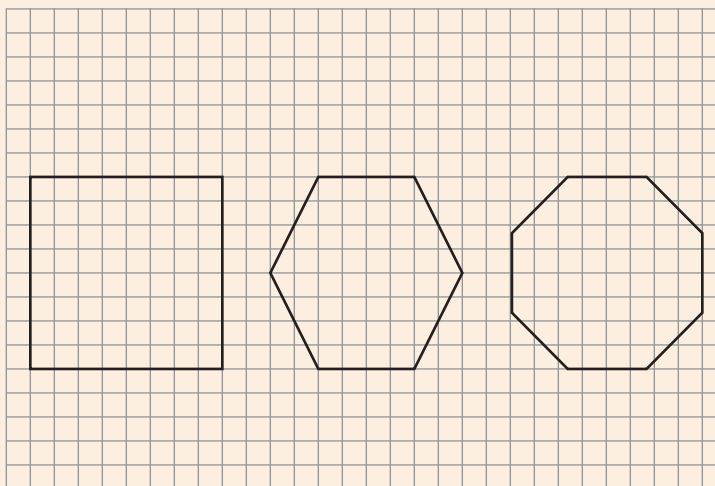
3. Do you think that a polygon with a greater number of sides has a greater perimeter than a polygon with a lesser number of sides? Why or why not? Use complete sentences to explain your reasoning.

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## Problem 3 Finishing the Deck



Before the homeowners decide which deck shape to use, they will consider the amount of space that each deck shape covers. Each square represents a square that is one foot long and one foot wide.



- A. How much space does the quadrilateral shape cover? Use complete sentences to explain how you found your answer.

### Problem 3 Finishing the Deck



- B. How much space does the hexagon shape cover? Use complete sentences to explain how you got your answer.
- C. How much space does the octagon shape cover? Use complete sentences to explain how you got your answer.
- D. Which plan would you choose based on your answers to part (A) through part (C)? Use complete sentences to explain your reasoning.

### Investigate Problem 3



1. In Problem 3, you found the **area**, or the number of square units covered by each deck shape. For which deck shape(s) was it the most difficult to find the area? Use complete sentences to explain your reasoning.



2. Complete the table below to show the perimeter and area of each deck shape. Area is measured in square units.

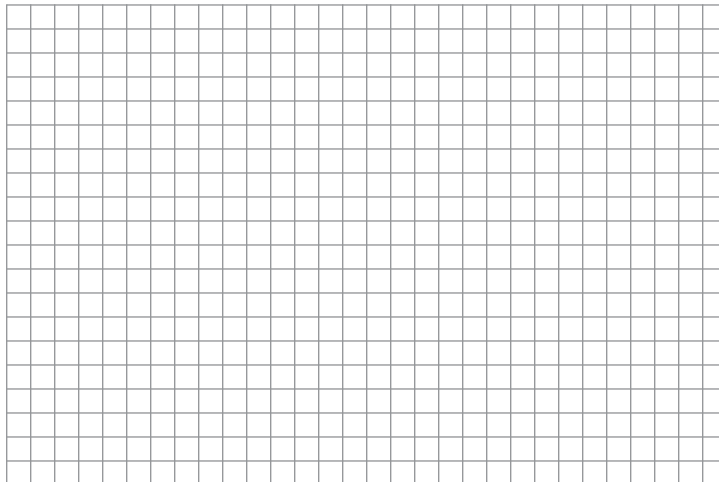
Deck shape	Quadrilateral	Hexagon	Octagon
Perimeter (feet)			
Area (square feet)			

3. Which deck shape would you choose based on the table in Question 2? Use complete sentences to explain your reasoning.

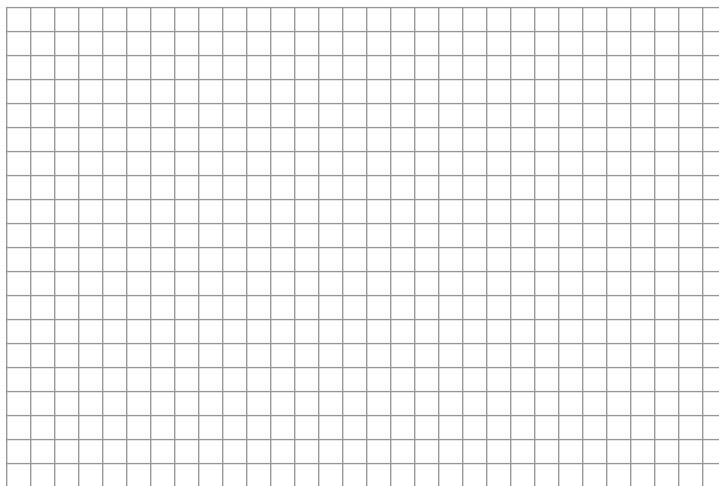
## Investigate Problem 3



4. If it is possible, draw two polygons that have the *same area* but *different perimeters* on the grid below. Then find the area and perimeter of your polygons.



5. If it is possible, draw two polygons that have the *same perimeter* but *different areas* on the grid below. Then find the area and perimeter of your polygons.



## Investigate Problem 3

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6. The homeowners decide on an octagonal deck, with all sides the same length and a perimeter of 128 feet. The perpendicular distance from the center of the deck to any side is about 19 feet. They will build the deck themselves. The product used to seal the wood surface of the deck is painted on and comes in one-gallon cans. Each gallon of sealing product covers 125 square feet. Draw a diagram of the problem situation. Then find the number of gallons of sealant that the homeowners will need to seal the deck with one and two coats. Show all your work and use complete sentences in your answer.

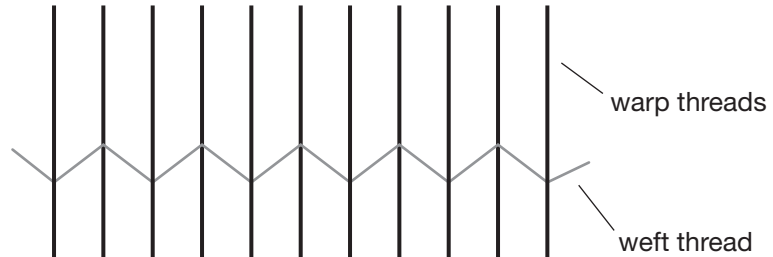
## Objectives

In this lesson, you will:

- Find areas of rectangles.
- Find perimeters of rectangles.
- Find unknown measures of rectangles.
- Find areas of parallelograms.
- Find unknown measures of parallelograms.



**SCENARIO** An artist creates rugs on a machine called a loom. On the loom, a piece of thread called a *weft* thread is woven through vertical pieces of thread called *warp* threads.



## Key Terms

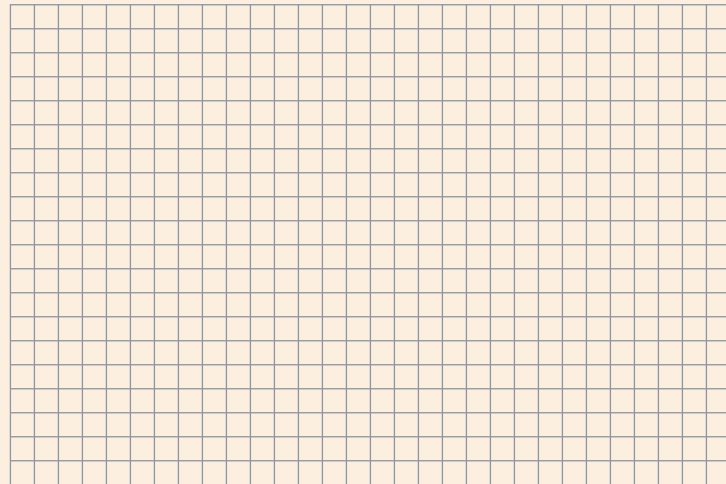
- rectangle
- length
- width
- base
- height
- parallelogram



### Problem 1 A Rectangular Rug

The artist is currently creating *rectangular* rugs. A **rectangle** is a quadrilateral that has four right angles.

- A.** One rectangular rug is seven feet long and three feet wide. Draw a model of this rug on the grid below. Each square on the grid represents a square that is one foot long and one foot wide. The area of one grid square is one square foot.



- B.** What is the area of this rug? Use a complete sentence in your answer.

How did you determine the area of the rug? Use a complete sentence in your answer.

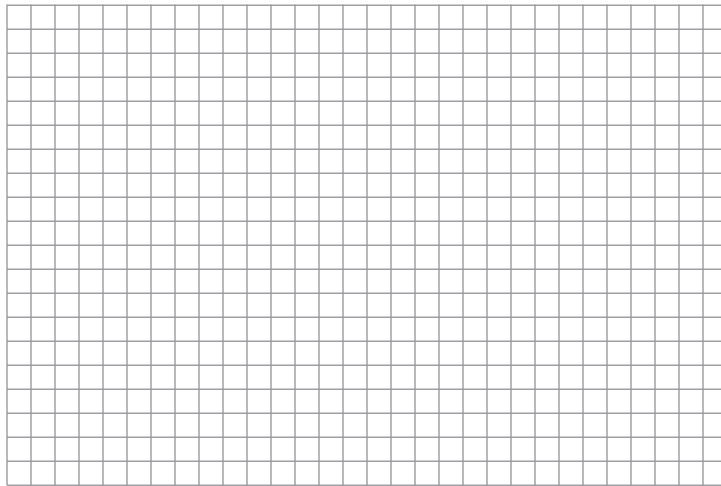
## Problem 1 A Rectangular Rug

- C. What is the perimeter of this rug? Use a complete sentence in your answer.

How did you determine the perimeter of the rug? Use a complete sentence in your answer.

## Investigate Problem 1

1. Draw six different rectangles on the grid below. Use the letters *A* through *F* to name each rectangle.



2. Complete the table below to show the length, width, area, and perimeter of each rectangle.

Rectangle	Length (units)	Width (units)	Perimeter (units)	Area (square units)
<i>A</i>				
<i>B</i>				
<i>C</i>				
<i>D</i>				
<i>E</i>				
<i>F</i>				

## Investigate Problem 1



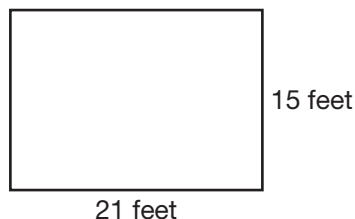
3. Can you determine the perimeter of a rectangle without drawing the rectangle if you know the rectangle's length and width? If so, explain how you can do this. Use complete sentences in your answer.

Write a formula that you can use to find the perimeter of any rectangle. Use  $\ell$  for the length of the rectangle,  $w$  for the width of the rectangle, and  $P$  for the perimeter. Make sure that the expression for the perimeter is in simplest form.

4. Can you determine the area of a rectangle without drawing the rectangle if you know the rectangle's length and width? If so, explain how you can do this. Use complete sentences in your answer.

Write a formula that you can use to find the area of any rectangle. Use  $\ell$  for the length of the rectangle,  $w$  for the width of the rectangle, and  $A$  for the area of the rectangle.

5. In each rectangle below, either the length, width, or area of the rectangle is unknown. Find the value of the unknown measure. Then find the perimeter. Use a complete sentence in your answer.

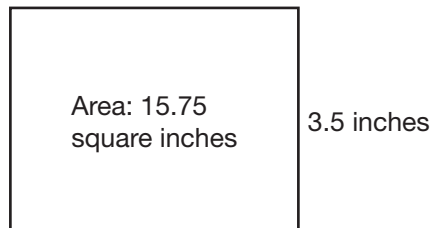


### Take Note

The length of a rectangle can also be called the **base** of the rectangle and the width of a rectangle can also be called the **height**.

## Investigate Problem 1

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6. Find the perimeter and area of a rectangle that is 11 meters long and 5 meters wide. Show all your work and use a complete sentence in your answer.

Now double the length and width of this rectangle. Find the perimeter of the new rectangle. Show all your work and use a complete sentence in your answer.

What effect does doubling the length and width have on the perimeter? Use a complete sentence in your answer.

### Investigate Problem 1

Do you think that doubling the length and width will have the same effect on the area? Use complete sentences to explain your reasoning.

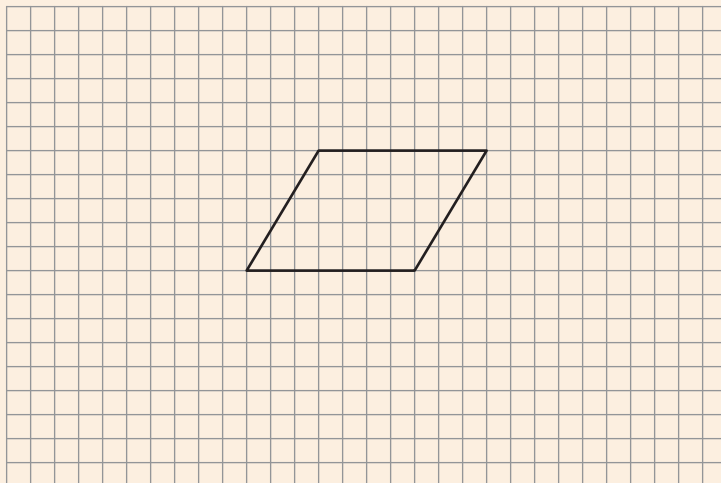
Find the area of the rectangle that is 22 meters long and 10 meters wide. Show all your work and use a complete sentence in your answer.

What effect does doubling the length and width have on the area? Use a complete sentence in your answer.

### Problem 2 A Parallelogram Rug



The artist has a special request from a client. The client would like a rug in the shape of a parallelogram. A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel. A model of the rug is shown on the grid below. Each square on the grid represents a square that is one foot long and one foot wide.



- A. Use complete sentences to explain how you can create a rectangle from the parallelogram above so that the rectangle and the parallelogram have the same area. Then check your answer by demonstrating your method on a separate sheet of grid paper. Draw your rectangle on top of the parallelogram above.

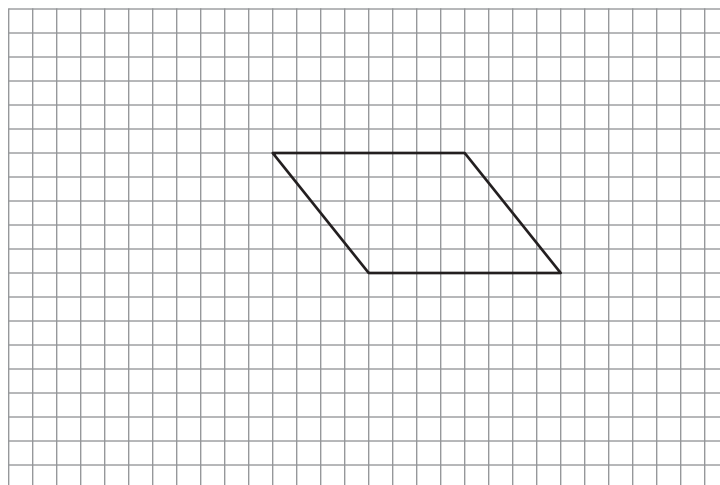
**Problem 2** A Parallelogram Rug

- B.** What is the area of the rectangle from part (A)? Show all your work and use a complete sentence in your answer.
- C.** What is the area of the rug? Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

**Investigate Problem 2**

1. The artist's client requests another parallelogram rug. A model of the new rug is shown on the grid below. Find the area of the rug. Use complete sentences to explain how you found your answer. Each square on the grid represents a square that is one foot long and one foot wide.



## Investigate Problem 2

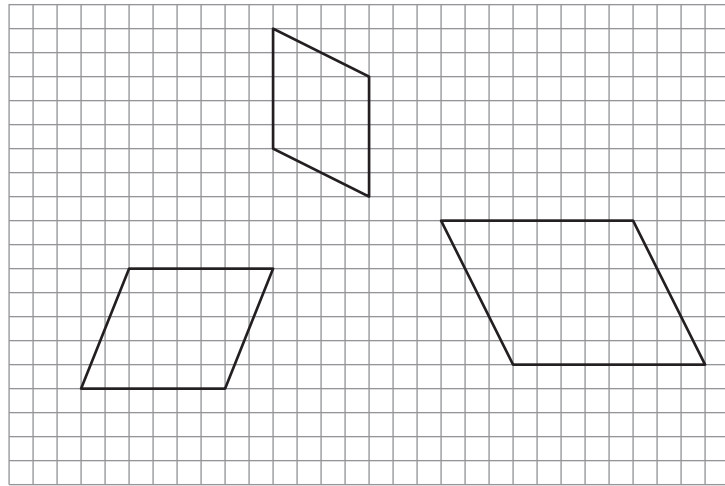


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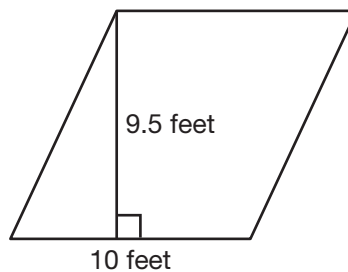
- 2. Just the Math: Height of a Parallelogram** The **height** of a parallelogram is the length of a perpendicular line segment that connects one of the corners of the parallelogram to the opposite side as shown below. The opposite side is called the **base** of the parallelogram.



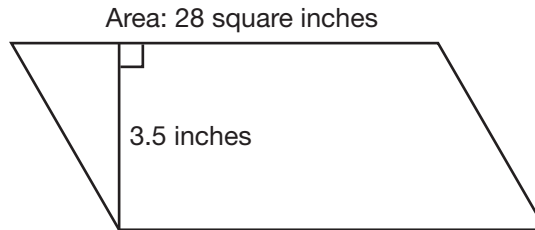
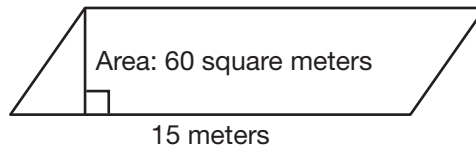
For each parallelogram, draw a segment that represents the height and label the height with its measure. Then label the base with its measure.



- 3.** Write a formula for the area of a parallelogram. Use  $b$  for the base of the parallelogram,  $h$  for the height, and  $A$  for the area.
- 4.** In each parallelogram below, either the length, width, or area of the parallelogram is unknown. Find the value of the unknown measure. Use a complete sentence in your answer.



## Investigate Problem 2



5. The artist charges \$20 per square foot of rug for a basic design. A client orders one basic rectangular rug that is 6 feet long and 4 feet wide and one basic rug in the shape of a parallelogram with a base that is 8 feet long and a height that is 3 feet. What is the total cost for the rugs? Show all your work and use a complete sentence in your answer.

## Objectives

In this lesson, you will:

- Find areas of triangles.
- Compare areas of triangles.
- Find perimeters of triangles.
- Find unknown measures of triangles.



**SCENARIO** In sailboat races, one of the typical shapes of a racing course is triangular. The course path is identified by buoys called marks. When the course is a triangle, the marks are located at the vertices of the triangle.

## Key Terms

- triangle
- vertex
- base
- height
- altitude

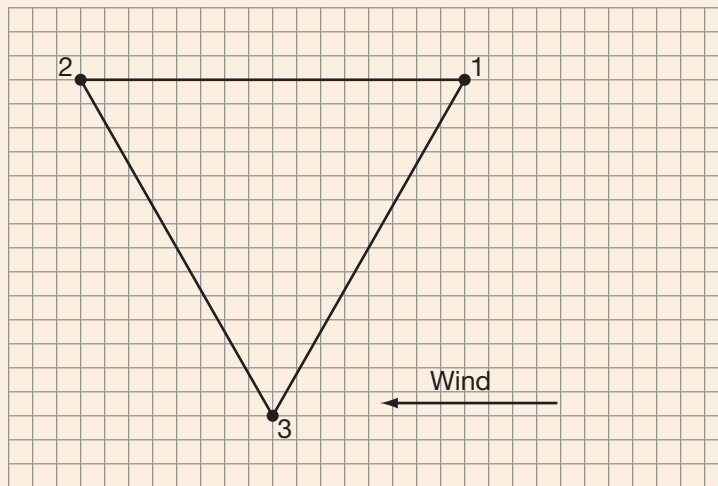


## Take Note

The **vertex** of a triangle is the point at which two sides of the triangle meet. The plural form of vertex is **vertices**.

## Problem 1 The Race Course

A sample course with the marks numbered is shown on the grid below. Each square on the grid represents a square that is one tenth of a kilometer long and one tenth of a kilometer wide.



- A. How many grid squares in a row create an area that is one kilometer long and one tenth of a kilometer wide? Use a complete sentence in your answer.
- B. How many grid squares are in an area that is one kilometer long and one kilometer wide? Use a complete sentence in your answer.
- C. Estimate the area enclosed by the course. Use a complete sentence in your answer.

## Problem 1 The Race Course

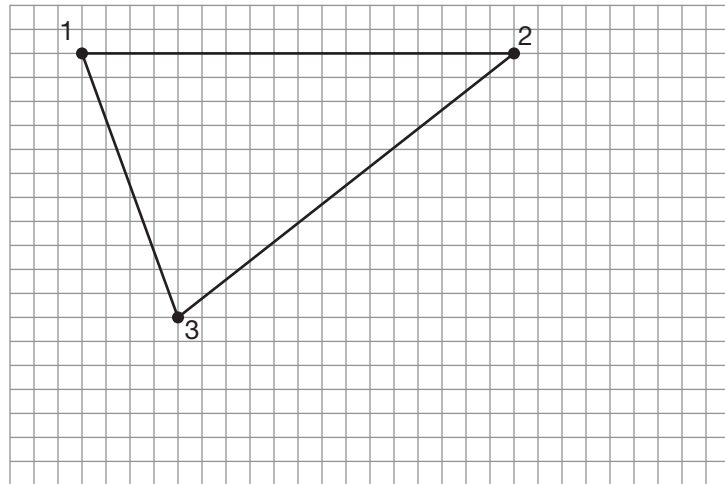
How did you determine the area? Use complete sentences in your answer.

- D.** Is your area from part (C) exact? Use a complete sentence to explain your reasoning.
- E.** On the grid on the previous page, use two sides of the triangle on the previous page to draw a parallelogram.
- F.** Find the area of the parallelogram. Show all your work and use a complete sentence in your answer.
- G.** Can you find the exact area of the triangle by using the area of the parallelogram? If so, explain why you can do this. Use complete sentences in your answer.
- H.** Find the exact area enclosed by the triangular course. Show all your work and use a complete sentence in your answer.
- I.** How does the area in part (H) compare to the area in part (C)? Use a complete sentence in your answer.

## Investigate Problem 1

1. In Problem 1, how does the area of the parallelogram relate to the area of the triangle? Use a complete sentence in your answer.

2. Consider the race course shown on the grid below. Each square on the grid represents a square that is one tenth of a kilometer long and one tenth of a kilometer wide. Find the area enclosed by the course. Show all your work and use a complete sentence in your answer.



3. What information about the triangle did you need to find the area in Question 2? Use complete sentences to explain your answer.

## Investigate Problem 1

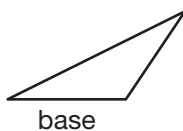
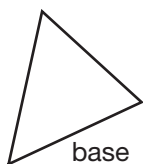
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### 4. Just the Math: Base and Height of a Triangle

In order to find the area of any triangle, you need to know the height of the triangle. The **height**, or **altitude**, of a triangle is the perpendicular distance from one of the vertices to the opposite side. This side is the **base** of the triangle.

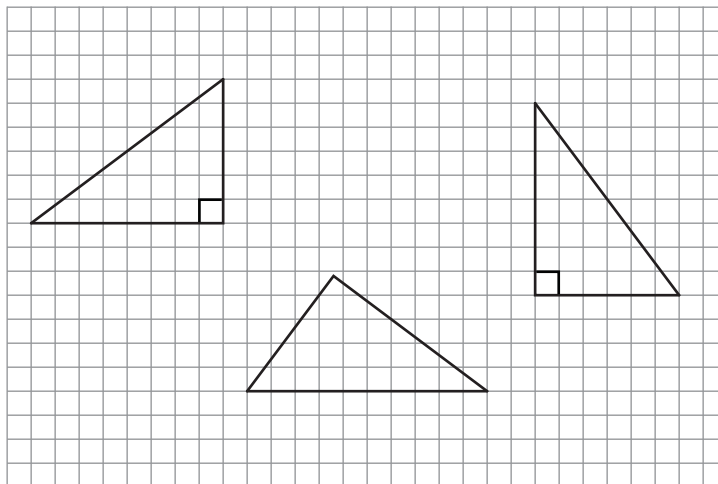
For each triangle below, one side is labeled as the base. Draw and label the segment that represents the triangle's height.



5. Can a triangle have more than one height? Use complete sentences to explain your reasoning.

6. Write a formula that you can use to find the area of any triangle. Use  $b$  for the length of the base,  $h$  for the height, and  $A$  for the area of the triangle.

7. The triangles below are all the same size and shape—they have just been rotated so that a different side is horizontal. The base of each triangle is the horizontal side. Label the base and height of each triangle with their measures.



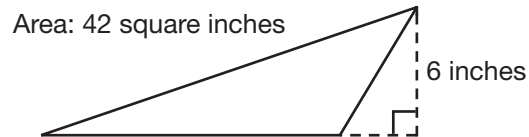
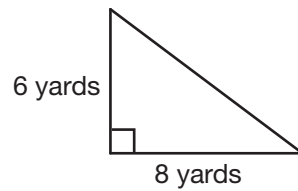
What do you know about the areas of the three triangles? Use a complete sentence to explain your reasoning.

## Investigate Problem 1

Use complete sentences to describe what happens to the height of a triangle as the length of the base changes if the area remains the same.

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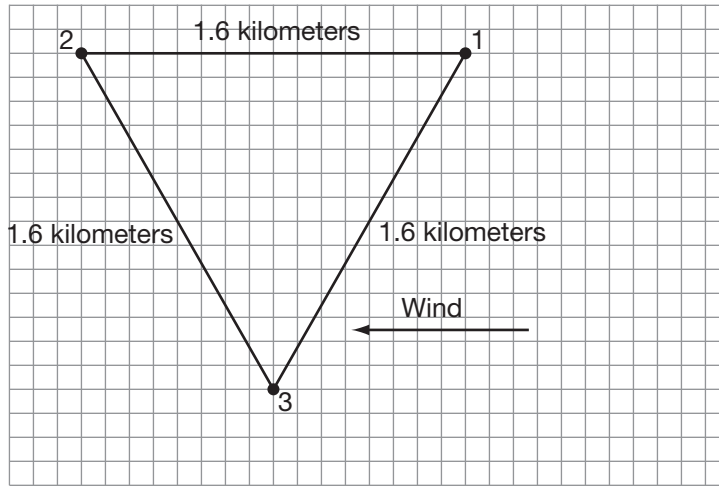
8. Find the unknown measure in each triangle below. Show all your work and use a complete sentence in your answer.



## Investigate Problem 1

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9. The original race course is shown below, but now the lengths of the legs of the race are given. If a boat must complete the course once, how long is the race? Show all your work and use a complete sentence in your answer.



What geometric name is given to this measurement?  
Use a complete sentence in your answer.

It is not uncommon for a boat to have to go around a course more than once or revisit a leg of the course more than once. Suppose that to complete the race, a boat has to sail to the marks in the following order: 1, 2, 3, 1, 3, 1, 2, 3, 1, 3. How long is this race? Show all your work and use a complete sentence in your answer.

If a boat is running the race, do you think that the boat will travel more than or less than the race length you found above? Use complete sentences to explain your reasoning.

**Objectives**

In this lesson, you will:

- Find areas of trapezoids.
- Find unknown measures of trapezoids.

**Key Terms**

- trapezoid
- base
- height



**SCENARIO** In most classrooms, a projection screen is hung above the blackboard. Because the screen is located above where the projector usually sits, the projector has to be tilted upward so that the image from the projector appears on the screen. This tilting can cause “keystoning,” which is a distortion of the image. A normal image and possible distorted image are shown below.

Things to remember when interviewing for a job

- Dress neatly
- Arrive early
- Be polite



Things to remember when interviewing for a job

- Dress neatly
- Arrive early
- Be polite

**Problem 1****How Distorted?**

- A.** Describe how the normal image has been distorted. Use complete sentences in your answer.
  
- B.** Describe the shapes formed by the normal image and the distorted image. Use complete sentences in your answer.
  
- C.** Which image do you think has a larger area? Use complete sentences to explain your reasoning.

## Problem 1 How Distorted?



- D. The normal image and the distorted image are shown on the grid below. Each square on the grid represents a square that is four inches long and four inches wide. Find the area of each image and write it in the center of the image.



- E. How do the areas of the images compare? Use a complete sentence in your answer.
- F. Is your area of the distorted image exact? Use a complete sentence to explain your reasoning.

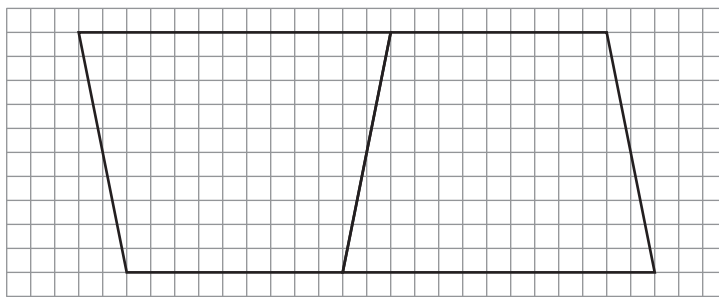
## Investigate Problem 1

1. Consider the distorted image in part (D). How can you use the area formulas you already know to find the exact area of this image? Use a complete sentence in your answer.
2. Find the exact area of the distorted image. Show all your work and use a complete sentence in your answer.

## Investigate Problem 1

1

- How do the exact areas of the normal image and the distorted image compare? Use a complete sentence in your answer.
- Consider the distorted image again. Suppose that we make an exact copy of this image, flip it vertically, and move it next to the image as shown below.



What is the geometric figure that is formed from these images? Use a complete sentence in your answer.

Find the area of the geometric figure. Then find the area of the distorted image. Show all your work and use a complete sentence in your answer.

- Was it easier to find the area of the distorted image by using your method in Question 1 or by using the method above in Question 4? Use complete sentences to explain your reasoning.



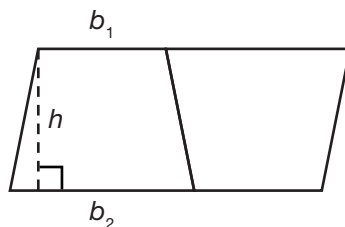
- Just the Math: Trapezoid** The distorted image is a *trapezoid*. A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called the **bases** of the trapezoid. The **height** of a trapezoid is the perpendicular distance between the bases. Identify the lengths of the bases and the height of the distorted image.

## Investigate Problem 1

1



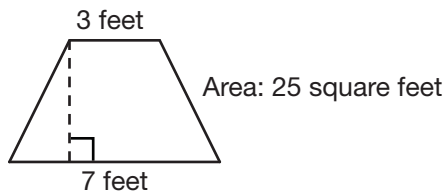
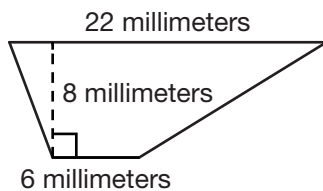
7. Consider the trapezoid below on the left. Suppose that we make an exact copy of this trapezoid and move it next to the trapezoid as shown below. Label the bases of the trapezoid on the right.



Write an expression for the area of the entire figure.

Write an expression for the area of one of the trapezoids.  
Use a complete sentence to explain how you got your answer.

8. In each trapezoid, either the height, the length of one base, or the area is unknown. Determine the value of the unknown measure. Use a complete sentence in your answer.

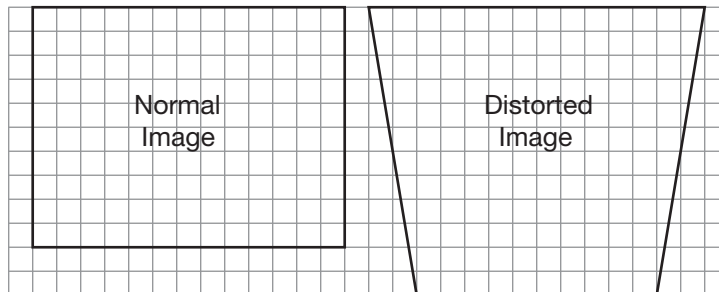


## Investigate Problem 1



1

9. The projector in Problem 1 was tilted differently to create the distorted image shown below. Each square on the grid represents a square that is four inches long and four inches wide.



What is the area of the distorted image? Show all your work and use a complete sentence in your answer.

How does the area of the distorted image compare to the area of the normal image? Use a complete sentence in your answer.

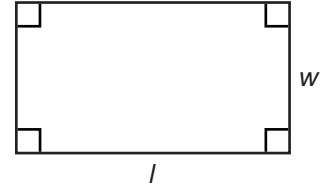
## Summary

## Area Formulas

In this lesson and the last two lessons, you came up with the following area formulas for common geometric figures:

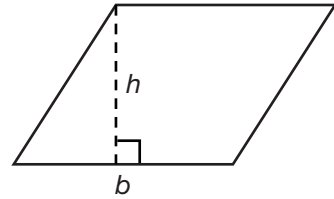
- Area of a Rectangle:

$$A = \ell w$$



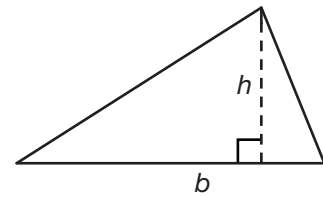
- Area of a Parallelogram:

$$A = bh$$



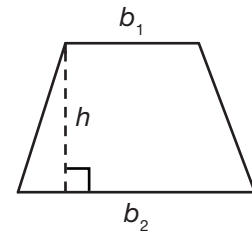
- Area of a Triangle:

$$A = \frac{1}{2}bh$$



- Area of a Trapezoid:

$$A = \frac{1}{2}(b_1 + b_2)h$$



**Objectives**

In this lesson, you will:

- Find areas of regular polygons.



**SCENARIO** Have you ever noticed that every stop sign looks exactly the same, every yield sign looks exactly the same, and so on? This is because the Federal Highway Administration has standards that indicate the exact sizes and colors of roadway signs. Most of the sign shapes are polygons.

**Key Terms**

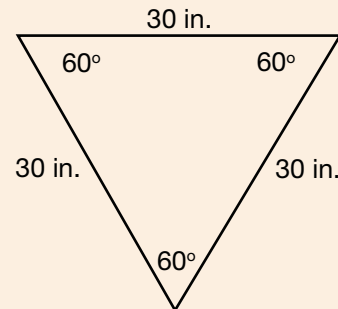
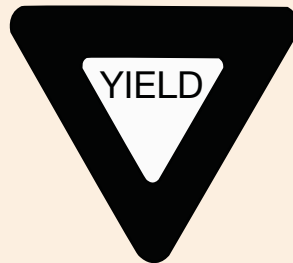
- regular polygon
- apothem

**Take Note**

An **angle** is formed where two sides of a polygon come together. Angles are measured in degrees. The symbol for degrees is  $^{\circ}$ .

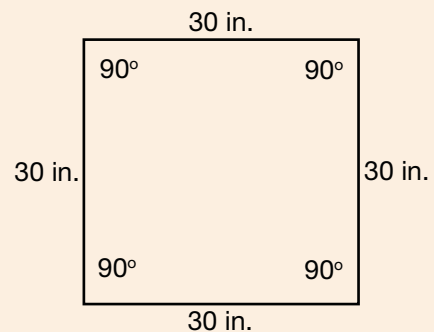
**Problem 1** How Big is that Sign?

- A. The specifications for the smallest possible yield sign are shown below.



What is special about the triangle that forms the yield sign? Use a complete sentence in your answer.

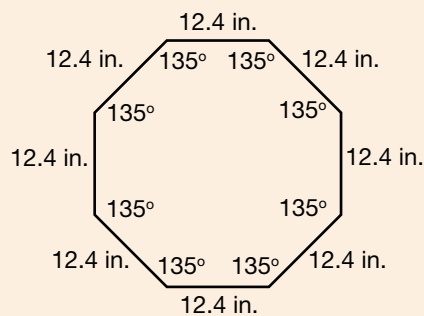
- B. The specifications for a “Do Not Enter” sign are shown below.



What is special about the quadrilateral that forms the “Do Not Enter” sign? Use a complete sentence in your answer.

## Problem 1 How Big is that Sign?

C. The specifications for a stop sign are shown below.

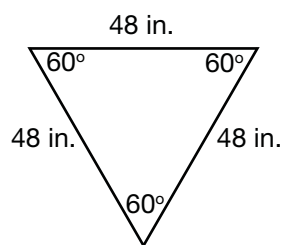
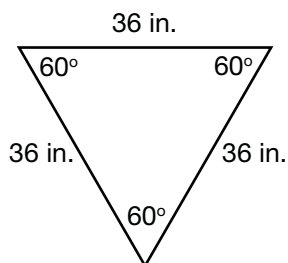


What is special about the octagon that forms the stop sign? Use a complete sentence in your answer.

## Investigate Problem 1

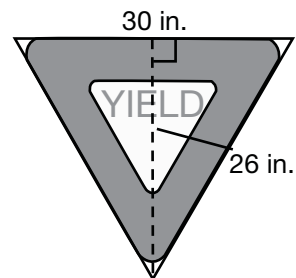


1. **Just the Math: Regular Polygons** The polygons in Problem 1 are special polygons called *regular polygons*. A **regular polygon** is a polygon in which all sides are equal in length and all angles are equal in measure. Two other possible sizes for a yield sign are shown below.



Are these signs regular polygons? What can you conclude about all regular triangles? Use complete sentences in your answer.

2. The yield sign from part (A) is shown with its approximate height. Find the area of this yield sign. Show all your work and use a complete sentence in your answer.



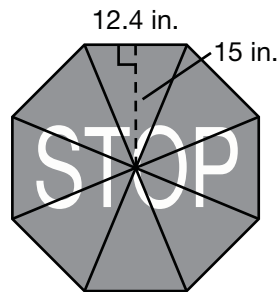
## Investigate Problem 1

3. Find the area of the “Do Not Enter” sign from part (B). Show all your work and use a complete sentence in your answer.

### Take Note

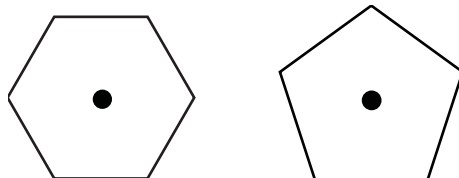
When two polygons are exactly the same size and exactly same shape, the polygons are said to be **congruent**.

4. To find the area of the stop sign from part (C), we can use the fact that a regular polygon can be divided into triangles that are all exactly the same size and exactly the same shape. The bases of the triangles are the sides of the polygon as shown below. In this case, the height of each triangle is approximately 15 inches. Find the area of the stop sign. Round your answer to the nearest tenth if necessary. Show all your work and use a complete sentence in your answer.



Use complete sentences to explain how you found the area of the stop sign.

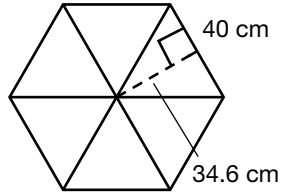
5. **Just the Math: Apothem** The height of the triangle in the stop sign in Question 4 is the *apothem* of the octagon. The **apothem** of a regular polygon is the perpendicular distance from the center of the regular polygon to a side of the regular polygon. Draw a segment that represents an apothem on each regular polygon shown below. The center of the polygon is marked by a point.



## Investigate Problem 1

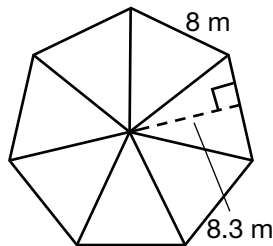
1

6. The hexagon shown below is a regular hexagon. Find the area of the hexagon. Show all your work and use a complete sentence in your answer.



Use a complete sentence to explain how you found the area of the regular hexagon.

7. The heptagon shown below is a regular heptagon. Find the area of the heptagon. Show all your work and use a complete sentence in your answer.



Use a complete sentence to explain how you found the area of the regular heptagon.

8. Explain how you can find the area of a regular polygon if you know the length of the apothem and the length of each side. Use complete sentences in your answer.

## Investigate Problem 1

9. Write a formula for the area of a regular polygon with  $n$  sides. Use  $a$  for the length of the apothem and  $\ell$  for the length of one side of the polygon.
  
10. The length of one side of a regular nonagon is 24 feet and the length of the apothem is approximately 33 feet. Find the area of the regular nonagon. Show all your work and use a complete sentence in your answer.
  
11. The side length of the largest possible stop sign is 20 inches and the length of the apothem is approximately 24.1 inches. What is the area of the largest possible stop sign? Show all your work and use a complete sentence in your answer.

The side length of the smallest possible stop sign is 9.9 inches and the length of the apothem is approximately 12 inches. What is the area of the smallest possible stop sign? Show your work and use a complete sentence in your answer.

How many times larger is the area of largest possible stop sign than the area of the smallest possible stop sign? Show all your work and use a complete sentence in your answer.



## Objectives

In this lesson, you will:

- Find circumferences of circles.
- Find areas of circles.
- Find unknown measures of circles.



**SCENARIO** The amount of light that comes into a camera lens is controlled by the *aperture* of the lens. Your pupil functions as the aperture of your eye. The aperture of a lens has a circular shape, and its size can change like the size of your pupil changes.

## Key Terms

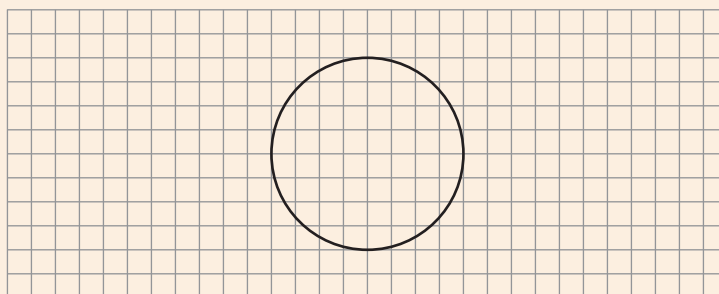
- diameter
- circumference
- radius
- irrational number
- area



## Problem 1

### The Distance Around the Aperture

- A. One size of an aperture for a camera lens is shown on the grid below. Each square on the grid represents a square that is 5 millimeters long and 5 millimeters wide.



Find the diameter of the aperture. Use a complete sentence in your answer.

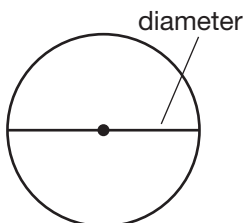
Then find the **circumference** of, or distance around, the circle. Use complete sentences to explain how you got your answer.

What is the ratio of the circumference to the diameter? Use a complete sentence in your answer.

Write this ratio as a decimal. Round your answer to two decimal places if necessary.

## Take Note

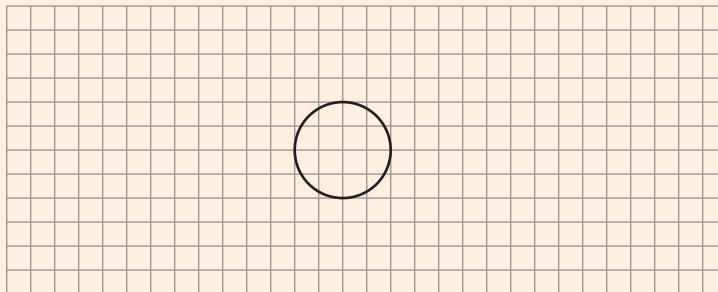
The **diameter** of a circle is the distance across the circle through the circle's center.



## Problem 1 The Distance Around the Aperture

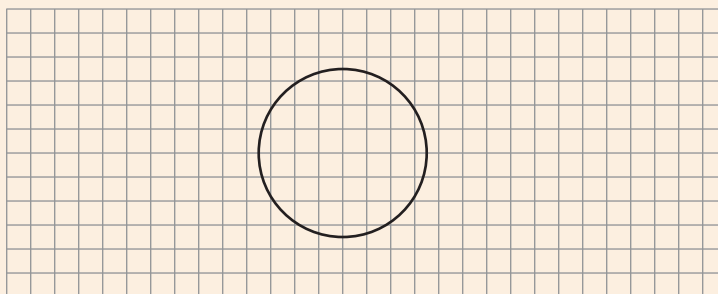
1

- B.** Another size of a camera lens aperture is shown below. Each square on the grid represents a square that is 5 millimeters long and 5 millimeters wide. Find the diameter and circumference of the aperture. Use a complete sentence in your answer.



Write the ratio of the circumference to the diameter as a decimal. Round your answer to two decimal places if necessary.

- C.** Another size of a camera lens aperture is shown below. Each square on the grid represents a square that is 5 millimeters long and 5 millimeters wide. Find the diameter and circumference of the aperture. Use a complete sentence in your answer.



Write the ratio of the circumference to the diameter as a decimal. Round your answer to two decimal places if necessary.

## Problem 1

### The Distance Around the Aperture

- D. How do the ratios in part (A) through part (C) compare to each other? Use a complete sentence in your answer.

## Investigate Problem 1



1. **Just the Math: Pi** It turns out that the ratio of a circle's circumference to its diameter is the same for all circles. Because the value of this ratio is an **irrational number** (its decimal representation neither repeats nor terminates), we use the symbol  $\pi$  (read as "pi") to represent the exact value of this ratio. Complete the statement below that defines  $\pi$  as a ratio of measures of a circle.

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

In this course, you will find that it is useful to use the fraction  $\frac{22}{7}$  to approximate  $\pi$  or the decimal 3.14 to approximate  $\pi$ .

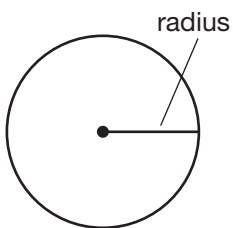
2. Write a formula that you can use to find the circumference of a circle when you know the diameter. Use  $C$  for the circumference and use  $d$  for the diameter.

Now write a formula that you can use to find the circumference of a circle when you know the radius. Use  $C$  for the circumference and use  $r$  for the radius.

3. Use either of your formulas above to find the circumferences of the apertures in part (A) through part (C). Show all your work and use a complete sentence in your answer. Use 3.14 for  $\pi$ .

### Take Note

The **radius** of a circle is one half of the diameter. The plural form of radius is radii.



## Investigate Problem 1

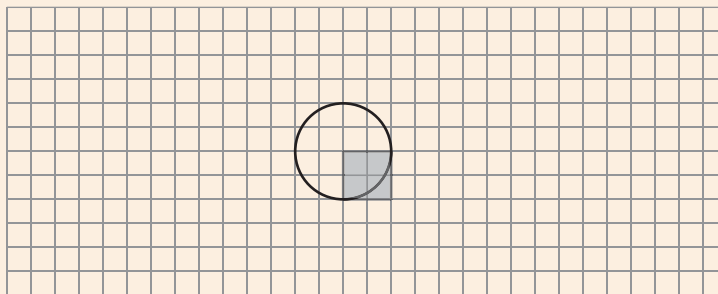
4. Suppose that the radius of an aperture is 5 millimeters. What is the circumference of the aperture? Show all your work and use a complete sentence in your answer. Leave your answers in terms of  $\pi$ .

Suppose that the radius is increased by one millimeter. Do you think that the circumference of the aperture will be increased by one millimeter? If so, justify your answer. If not, how does the circumference change? Use complete sentences in your answer.

Suppose that the radius is doubled to be 10 millimeters. Do you think that the circumference of the aperture will be doubled? If so, justify your answer. If not, how does the circumference change? Use complete sentences in your answer.

## Problem 2 The Area Inside the Aperture

- A. Consider the aperture shown below. Each square on the grid represents a square that is 5 millimeters long and 5 millimeters wide.



What is the diameter of the aperture? What is the radius?

## Problem 2 Ratios in the News



- B.** Consider the shaded square shown in part (A). How does the side length of the square relate to the circle? Use a complete sentence in your answer.

What is the area of this square? Use a complete sentence in your answer.

- C.** Estimate the number of these squares that it will take to cover the circle.
- D.** What is the estimated area of the aperture? Show all your work and use a complete sentence in your answer.

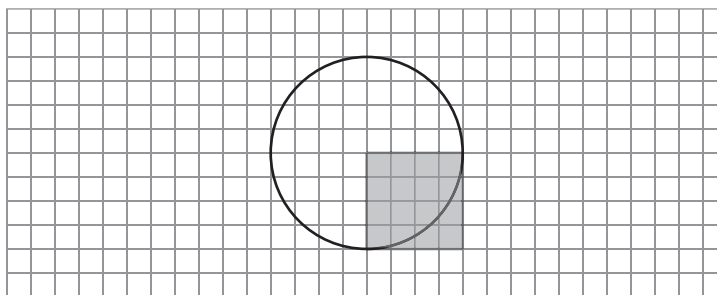
### Take Note

Whenever you are asked to make an estimate, you should use the words “about” or “approximately” to indicate that your answer is not exact.



## Investigate Problem 2

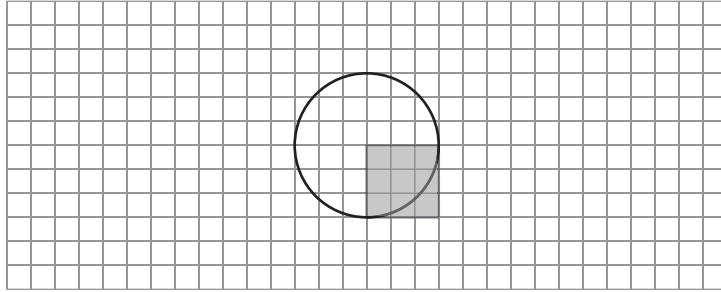
- 1.** Another camera lens aperture is shown below. Each square on the grid represents a square that is 5 millimeters long and 5 millimeters wide. Use the method shown in part (A) through part (D) of Problem 2 to estimate the area of the aperture. Show all your work and use a complete sentence in your answer.



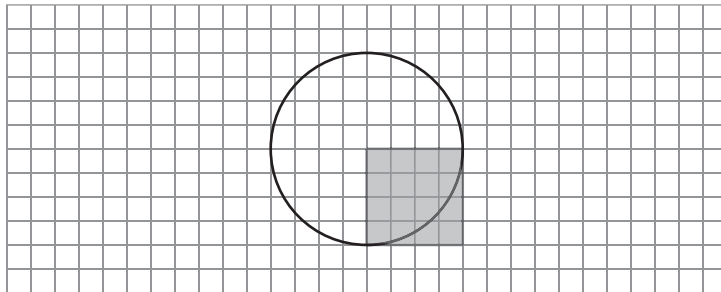
## Investigate Problem 2

1

2. Another camera lens aperture is shown below. Each square on the grid represents a square that is 10 millimeters long and 10 millimeters wide. Estimate the area of the aperture. Show all your work and use a complete sentence in your answer.



3. Another camera lens aperture is shown below. Each square on the grid represents a square that is 10 millimeters long and 10 millimeters wide. Estimate the area of the aperture. Show all your work and use a complete sentence in your answer.



## Investigate Problem 2

4. Complete the table below that shows the results from Problem 2.

Radius (millimeters)	Area of shaded square (square millimeters)	Number of shaded squares needed to cover circle	Estimated area of circle
10			
20			
30			
40			

5. What do you notice about the number of shaded squares needed to cover each circle in the table above? Use a complete sentence in your answer.



For each circle, what is the relationship between the area of the radius square and the estimated area of the circle? Use a complete sentence in your answer.

Is the number of shaded squares close to a number that you know? If so, name the number.

6. **Just the Math: Area of a Circle** In Problem 2, you discovered that the area  $A$  of a circle with a radius  $r$  is given by  $A = \pi r^2$ . Use the formula to find the areas of the apertures described in Question 4. Show all your work. Use 3.14 for  $\pi$ .

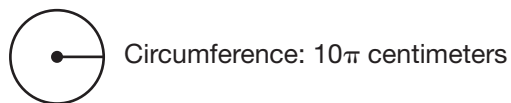
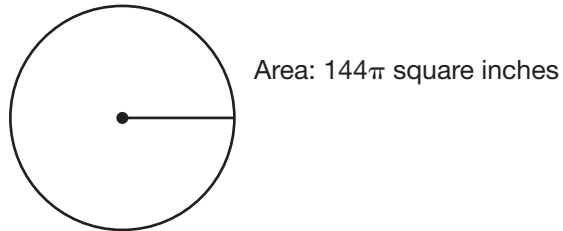
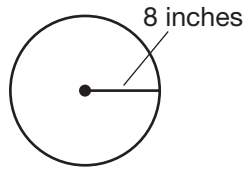


## Investigate Problem 2

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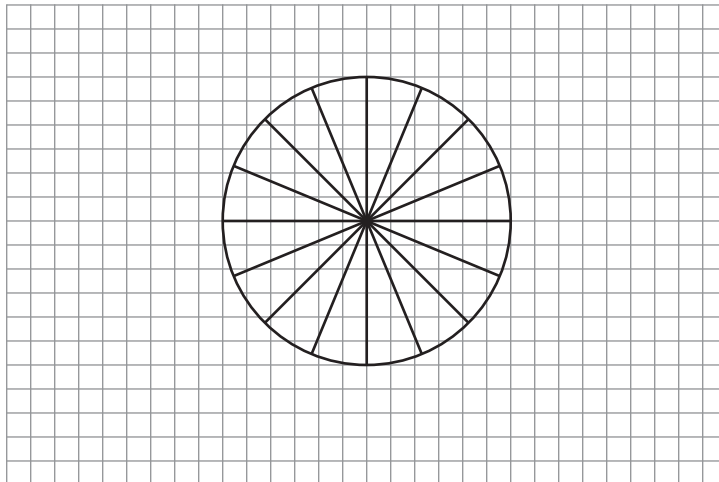


7. In each of the following circles you know one measure. Find the unknown measures: the radius, diameter, area, and/or circumference. Use a complete sentence in your answer. Leave your answers in terms of  $\pi$ .



## Investigate Problem 2

8. On a piece of graph paper, draw a large circle and divide it into 16 equal sections as shown below. What are the radius and circumference of your circle? Leave your answer in terms of  $\pi$ .



Now cut out each of the sections and line up the sections beside each other, turning every other piece upside down. Tape your shape onto a piece of paper. What is the approximate shape of the figure?

9. Find the base and height of your figure. Give your answer in terms of the radius and circumference of your circle. Use a complete sentence in your answer.
10. Find the area of your figure. What does this area represent?

Does your result confirm the formula for the area of a circle?

11. In Problem 2, you used two different methods for estimating the area of a circle. Which method do you prefer? Use complete sentences to explain your reasoning.



**Objectives**

In this lesson, you will:

- Find areas of composite figures.



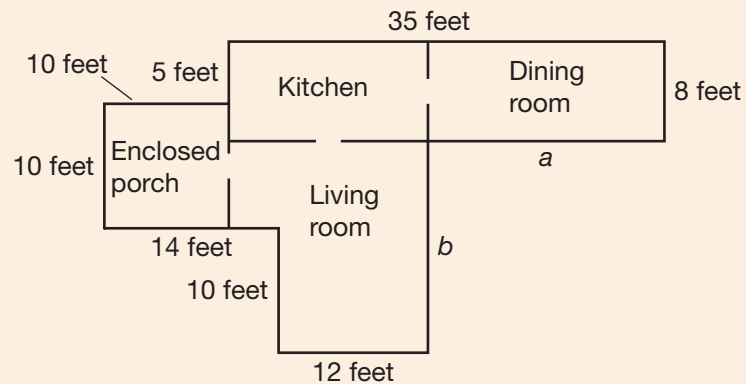
**SCENARIO** Most companies that do home renovations and repairs will send out a person to record measurements that will be needed for the job. These measurements are needed to estimate the cost of the job and to order the materials needed to complete the job.

**Key Terms**

- composite figures

**Problem 1****A Brand New Floor**

A carpeting company has been hired to install flooring on the first floor of a home. A diagram of this first floor is shown below.



- A.** Find the unknown lengths  $a$  and  $b$ . Show all your work and use a complete sentence in your answer.
- B.** The home owners would like to install indoor/outdoor carpeting on the enclosed porch. How many square feet of indoor/outdoor carpeting will be needed? Show all your work and use a complete sentence in your answer.

## Problem 1

### A Brand New Floor

**C.** The home owners would like to install loop carpeting in the living room. How many square feet of loop carpeting will be needed? Show all your work and use a complete sentence in your answer.

**D.** The home owners would like to install tile in the kitchen. How many square feet of tile will be needed? Show all your work and use a complete sentence in your answer.

**E.** The home owners would like to install wood flooring in the dining room. How many square feet of wood flooring will be needed? Show all your work and use a complete sentence in your answer.

## Investigate Problem 1



1. What is the total area of the first floor? Show all your work and use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to Question 1.

2. An employee from the carpeting company must now find the total cost of the materials for the job. Wood flooring and tile are sold by the square foot, but carpeting is sold by the square yard. So the employee must rewrite some of his areas in square yards. Draw a square that models one square foot. Use one inch to represent one foot.

There are three feet in one yard. Draw a square that models one square yard. Use one inch to represent one foot.

## Investigate Problem 1

Divide your square on the previous page into square feet. How many square feet are there in one square yard? Use a complete sentence in your answer.

1



3. Write the area of the enclosed porch in square yards by completing the expression below. Round your answer to the nearest tenth.

$$\boxed{\phantom{000}} \times \frac{1 \text{ square yard}}{9 \text{ square feet}} \approx \boxed{\phantom{00}} \text{ square yards}$$

What is the area of the enclosed porch in square yards?  
Use a complete sentence in your answer.

Use a complete sentence to explain how you can write an area in square yards that is written in square feet.



4. What is the area of the living room in square yards? Show all your work and use a complete sentence in your answer. Round your answer to the nearest tenth.

5. The indoor/outdoor carpeting costs \$10.75 per square yard, the loop carpeting costs \$7.50 per square yard, the wood flooring costs \$4.50 per square foot, and the tile costs \$4.25 per square foot. Find the total cost of the materials needed for the job. Show all your work and use a complete sentence in your answer.

## Investigate Problem 1

Do you think that the total cost is accurate? Why or why not? Use complete sentences in your answer.

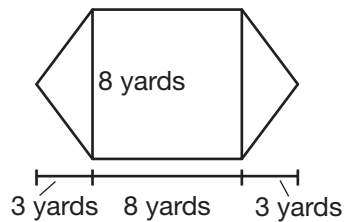
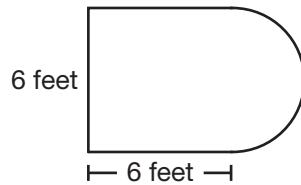
1



6. After the home owners see the total cost of the flooring for the job, they decide that the wood flooring is too expensive and decide to buy the same loop carpeting in the dining room as will be used in the living room. Find the total cost of the flooring for the first floor if the home owners decide to buy the loop carpeting for the dining room. Show all your work and use a complete sentence in your answer.



7. Find the area of the figure. Show all your work and use a complete sentence in your answer. Use 3.14 for  $\pi$ .



## Investigate Problem 1

1



8. Find the area of the shaded portion of the figure. Show all your work and use a complete sentence in your answer. Use 3.14 for  $\pi$ .

