

## Objectives

In this lesson, you will:

- Determine the scale factor in a dilation.
- Use a compass and a straightedge to perform a dilation.
- Perform dilations in the coordinate plane by using coordinate notation.

## Key Terms

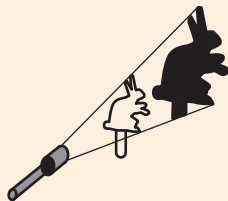
- ratio
- dilation
- center of dilation
- scale factor



**SCENARIO** You have volunteered to help at the children's booth at an art festival. The children that visit the booth will be able to create objects like animals or people out of poster board and popsicle sticks and then use a flashlight to create shadow puppets. Your job is to show the children how to use a flashlight and a wall to make their own puppet show.

## Problem 1 Making a Shadow

After some children have finished making their poster board animals or people, you show them how to use the flashlight to create a shadow puppet.



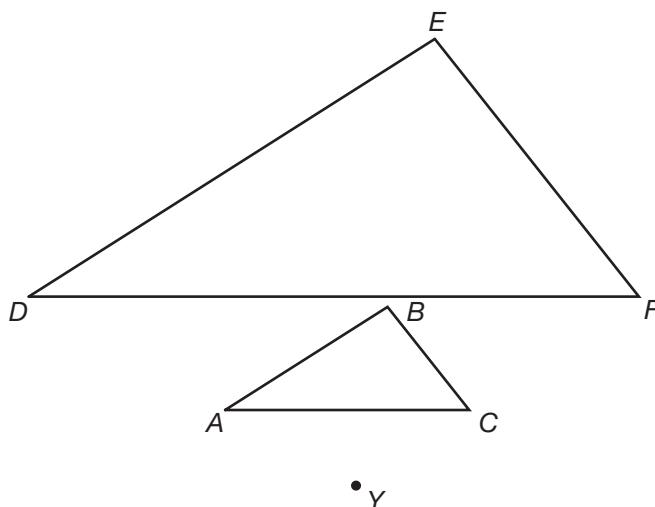
- A. How does the size of the shadow puppet compare to the size of the object made out of poster board and popsicle sticks? Use a complete sentence in your answer.
- B. How does the shape of the shadow puppet compare to the shape of the object made out of poster board and popsicle sticks? Use a complete sentence in your answer.
- C. Do you think that the process of creating a shadow puppet by using a flashlight on an object cut out of poster board is a transformation of the object? Why or why not? Use complete sentences in your explanation.

If so, do you think that the transformation is an isometry? Why or why not? Use complete sentences in your explanation.

## Investigate Problem 1



1. Consider  $\triangle ABC$ ,  $\triangle DEF$ , and point  $Y$ . Imagine that point  $Y$  is the flashlight and  $\triangle DEF$  is the shadow of  $\triangle ABC$ .



Draw the following line segments on the figure above. These line segments show you the path of the light from the flashlight.

$\overline{YD}$ ,  $\overline{YE}$ ,  $\overline{YF}$

What do you notice about the line segments? Use a complete sentence in your answer.

2. Use a metric ruler to find the following lengths. Find each length to the nearest tenth of a centimeter.

$YA =$  \_\_\_\_\_       $YD =$  \_\_\_\_\_

$YB =$  \_\_\_\_\_       $YE =$  \_\_\_\_\_

$YC =$  \_\_\_\_\_       $YF =$  \_\_\_\_\_

3. Complete the following ratios. Then write each ratio as a decimal.

$$\frac{YD}{YA} =$$

$$\frac{YE}{YB} =$$

$$\frac{YF}{YC} =$$

### Take Note

A **ratio** is a comparison of two numbers that uses division. The ratio of two numbers  $a$  and  $b$  ( $b$  cannot be zero), can be written in three ways.

$a$  to  $b$

$a : b$

$\frac{a}{b}$

## Investigate Problem 1

What do you notice about the ratios? Use a complete sentence in your answer.

Find the measures of the corresponding angles in the triangles. What do you know about the corresponding angles? Use complete sentences to explain your reasoning.



4. **Just the Math: Dilation** The shadow created by the flashlight is a kind of transformation called a *dilation*. In a **dilation**, the image is the same shape but a different size. What is the relationship between the image and preimage in a dilation? Use a complete sentence in your answer.

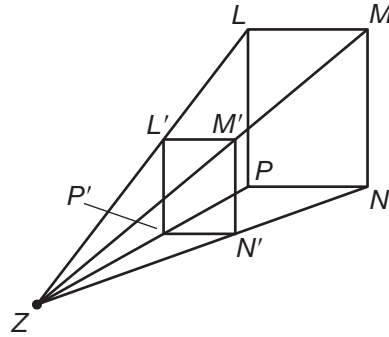
In any dilation, how will the corresponding angles in the image and preimage compare? Use a complete sentence in your answer.

In any dilation, how will the ratios of the lengths of the corresponding sides compare? Use a complete sentence in your answer.

5. Like rotations, dilations have a point that is the **center of dilation**. What is the center of the dilation shown in Question 1? Use a complete sentence in your answer.

## Investigate Problem 1

6. An image that is the result of a dilation is not always larger than the preimage, as shown below. Rectangle  $L'M'N'P'$  is a dilation of rectangle  $LMNP$ . The center of dilation is point  $Z$ .



Use the actual lengths, in centimeters, of the line segments to complete the ratios below. Then write each ratio as a decimal.

$$\frac{ZL'}{ZL} =$$

$$\frac{ZM'}{ZM} =$$

$$\frac{ZN'}{ZN} =$$

$$\frac{ZP'}{ZP} =$$

What do you notice about the ratios? Use a complete sentence in your answer.

7. **Just the Math: Scale Factor** The ratios in Question 3 and Question 6 are the **scale factors** of the dilations. When the scale factor is greater than 1, how does the image compare to the preimage? Use a complete sentence in your answer.

When the scale factor is less than 1, how does the image compare to the preimage? Use a complete sentence in your answer.

8. Complete the following statement.

The scale factor is the ratio of the distance from the center of dilation to a point on the \_\_\_\_\_ to the distance from the center of dilation to the corresponding point on the \_\_\_\_\_.

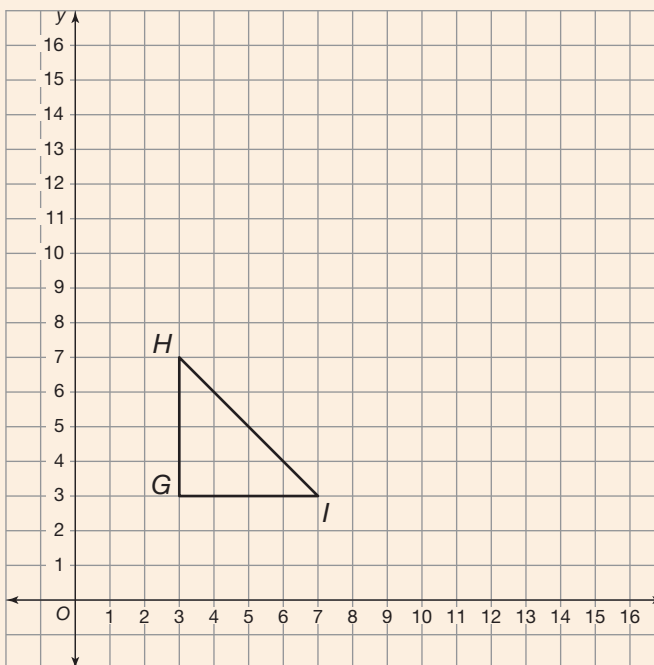
## Problem 2 Larger or Smaller?



You can use a compass and a straightedge to perform a dilation. Consider  $\triangle GHI$  shown on the grid below. You will dilate the triangle

by using the origin as the center and by using a scale factor of  $\frac{2}{1}$ .

How does the distance from the center of dilation to a point on the image of  $\triangle GHI$  compare to the distance from the center of dilation to a corresponding point on  $\triangle GHI$ ? Use a complete sentence to explain your reasoning.



- A. For each vertex of  $\triangle GHI$ , draw a ray that starts at the origin and passes through the vertex.
- B. Now place the point of the compass on the origin and open the compass until it reaches point G. This gives you  $OG$ . Find point  $G'$  by placing the point of the compass at point G and drawing an arc that intersects  $\overrightarrow{OG}$ . How does  $OG'$  compare to  $OG$ ? Use a complete sentence in your answer.

Find the rest of the vertices of the image in a similar way. Then draw line segments to form  $\triangle G'H'I'$ .

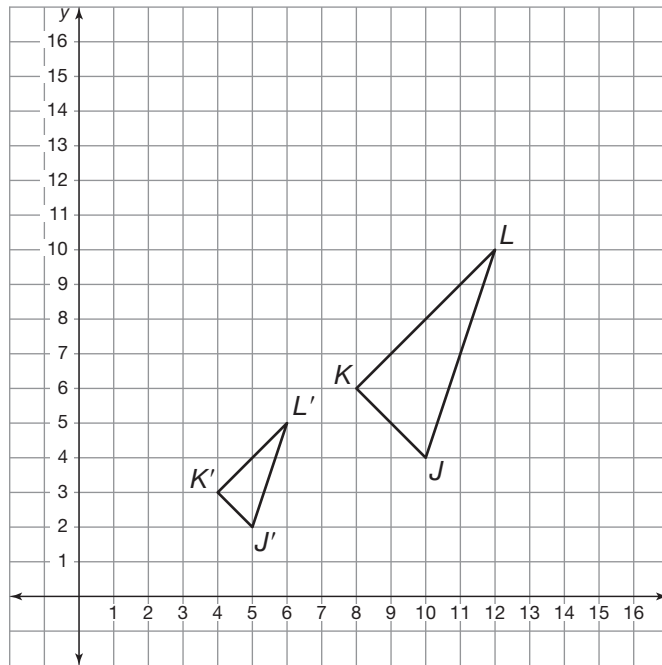
## Problem 2 Larger or Smaller?

- C. List the coordinates of the vertices of  $\triangle GHI$  and  $\triangle G'H'I'$ . How do the coordinates of the image compare to the coordinates of the preimage? Use complete sentences in your answer.

## Investigate Problem 2



1. Triangle  $J'K'L'$  is a dilation of  $\triangle JKL$ . The center of dilation is the origin.



List the coordinates of the vertices of  $\triangle JKL$  and  $\triangle J'K'L'$ . How do the coordinates of the image compare to the coordinates of the preimage? Use complete sentences in your answer.

## Investigate Problem 2

What is the scale factor of the dilation? Use complete sentences to explain how you found your answer.

How do you think you can use the scale factor to determine the coordinates of the vertices of an image? Use a complete sentence in your answer.

2. Complete the following statement.

The coordinate notation for a dilation with the center at the origin and a scale factor of  $k$  is

$(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$ .

3. The coordinates of the vertices of quadrilateral  $ABCD$  are  $A(-4, 2)$ ,  $B(-2, 8)$ ,  $C(4, 6)$ , and  $D(6, -8)$ . If quadrilateral  $ABCD$  is dilated when the center is at the origin and the scale factor is  $\frac{3}{2}$ , will the image be larger or smaller than quadrilateral  $ABCD$ ? Use a complete sentence to explain your reasoning.

Perform the dilation described above and find the vertices of quadrilateral  $A'B'C'D'$ . Show all your work and use a complete sentence in your answer.

