

## 2.2

# Turn Up the Volume

## Volume of a Prism

### Objectives

In this lesson, you will:

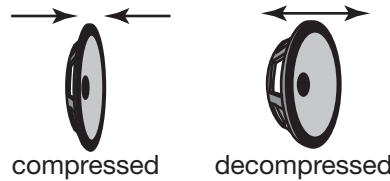
- Identify parts of prisms.
- Name prisms.
- Find volumes of prisms.

### Key Terms

- polyhedron
- faces
- prism
- bases
- lateral faces
- height
- volume



**SCENARIO** Sound is produced by a speaker when a flexible cone inside the speaker vibrates back and forth. The cone vibrates by compressing and decompressing.

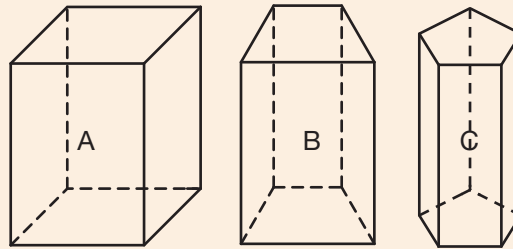


Because of this vibration, a speaker is usually enclosed in a rigid cabinet that can absorb these vibrations. Otherwise the sound coming out of the speaker would get drowned out by the sound of material being vibrated around the speaker, such as the floor upon which a speaker sits.

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### Problem 1 Speaker Design

Speaker cabinets come in a variety of sizes and shapes. The size and shape depends on how the speaker is to be used. A few speaker cabinet designs are shown below. The dashed lines indicate the shapes of the sides of the cabinets that cannot be seen.



- A.** The speaker cabinets above are *polyhedrons*. A **polyhedron** is a solid that is formed from polygons. The polygons that form the solid are called **faces**. How are the three cabinets similar? How are the three cabinets different? Use complete sentences in your answer.

## Problem 1 Speaker Design



- B.** Consider the polygons that form the tops of the speaker cabinets. What kind of polygon is each speaker cabinet top? Describe these polygons as precisely as possible. Use complete sentences in your answer.
- C.** Consider the polygons that form the sides of each speaker cabinet. What kind(s) of polygons are the sides of each speaker cabinet? Use a complete sentence in your answer.
- D.** Consider the polygons that form the bottoms of the speaker cabinets. What kind of polygon is each speaker cabinet bottom? Describe these polygons as precisely as possible. Use a complete sentence in your answer.
- E.** What do you notice about the top and bottom of each speaker cabinet? Use a complete sentence in your answer.

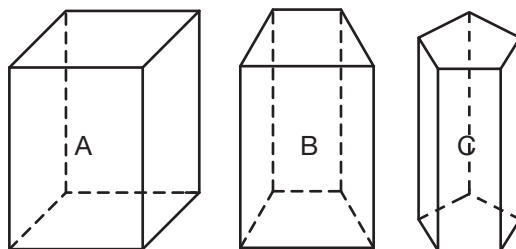
## Investigate Problem 1



### Take Note

Remember that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. So a rectangle is a special kind of parallelogram. A prism whose lateral faces are rectangles is called a **right prism** and a prism whose lateral faces are not rectangles is called an **oblique prism**. In this chapter, we will only be considering right prisms.

- 1. Just the Math: Prisms** The polyhedrons in Problem 1 are special polyhedrons called *prisms*. A **prism** is a polyhedron that has two congruent and parallel faces, called **bases**. The other faces of the prism are formed by joining the corresponding vertices of the bases. These faces are parallelograms and are called the **lateral faces** of the prism. The **height** of a prism is the length of a segment that is perpendicular to the bases.



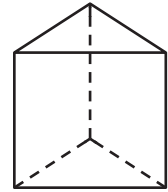
## Investigate Problem 1

Identify the number of bases and lateral faces that each prism in Question 1 has. Use complete sentences in your answer.

How is the number of lateral faces related to the base? Use a complete sentence in your answer.



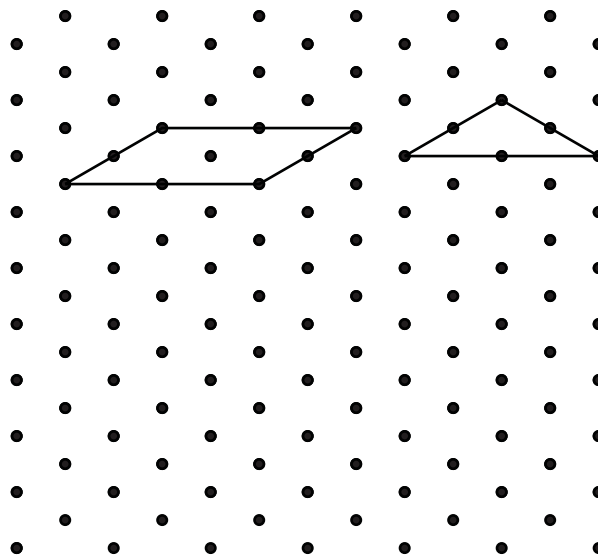
2. You can name prisms by the shape of their bases. For instance, the prism at the right is a triangular prism.



Identify the prisms in Question 1 by the shape of the bases.



3. You can use the dot grid below to draw your own prisms. The top bases of a rectangular prism and a triangular prism are already drawn for you. Complete each prism by drawing another base directly below the first base. You can go down the grid to your desired height. Then connect the corresponding vertices of the bases.



### Take Note

The rectangular base of the prism looks like a parallelogram because of the viewpoint. If you drew the rectangle with right angles, then you wouldn't be able to see all of the faces of the prism.

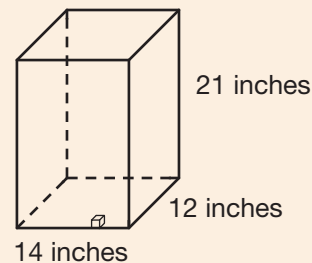
## Problem 2 The Space Inside the Speakers



The amount of space inside the speaker cabinet is important because it can affect the sound coming out of a speaker. Consider the speaker cabinet below. This cabinet is in the shape of a rectangular prism. A rectangular prism has three dimensions: length, width, and height.



- A. Imagine a layer of cubes on the bottom of the speaker cabinet shown. Each cube is one inch long, one inch wide, and one inch tall. How many cubes are in the layer? Use a complete sentence in your answer.



Use a complete sentence to explain how you found your answer to part (A).

- B. Suppose that you add another layer of cubes onto the existing layer. How many cubes are there in the two layers? Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (B).

- C. Suppose that you add another layer of cubes onto the first two layers. How many cubes are there in the three layers? Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (C).

### Take Note

If all of the faces of a right prism are congruent squares, then the prism is called a **cube**.

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## Problem 2 The Space Inside the Speakers

- D. How many cubes will it take to fill the speaker cabinet?  
Use a complete sentence in your answer.

Use a complete sentence to explain how you found your answer to part (D).

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### Investigate Problem 2



1. In part (A), how does the number of cubes in one layer relate to the area of the base of the prism? Use a complete sentence in your answer.



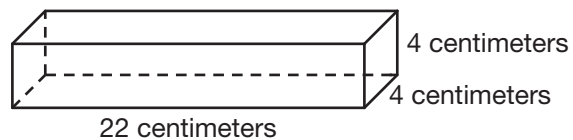
2. **Just the Math: Volume of a Prism** In Problem 2, you found the *volume* of, or the amount of space occupied by, the prism. You can find the area of any prism by multiplying the area of one of the bases by the height of the prism. This is written mathematically as

$$V = Bh$$

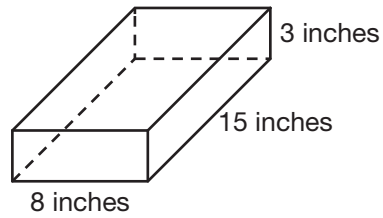
where  $V$  is the volume,  $B$  is the area of one of the prism's bases, and  $h$  is the prism's height. Volume is measured in cubic units because it is the product of three dimensions. What is the volume of the speaker cabinet in Problem 2? Use a complete sentence in your answer.

Write a formula for the volume of a rectangular prism with a length of  $\ell$  units, a width of  $w$  units, and a height of  $h$  units.

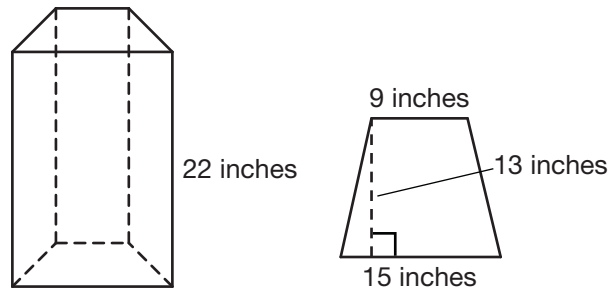
3. Find the volume of each rectangular prism. Show all your work and use a complete sentence in your answer.



## Investigate Problem 2



4. The speaker cabinet shown below is designed so that the sound is pushed outward. The dimensions of one of the bases are also shown. Describe the shape of the speaker cabinet. Use a complete sentence in your answer.



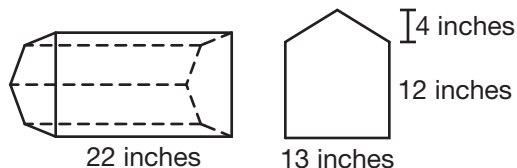
What is the area of a base of the speaker cabinet? Show all your work and use a complete sentence in your answer.

What is the height of the speaker cabinet? Use a complete sentence in your answer.

Find the volume of the speaker cabinet. Show all your work and use a complete sentence in your answer.

## Investigate Problem 2

5. The speaker cabinet shown below is designed so that the sound is pushed upward. The dimensions of one of the bases are also shown. Describe the shape of the speaker cabinet. Use a complete sentence in your answer.



What is the area of a base of the speaker cabinet? Show all your work and use a complete sentence in your answer.

How did you find the area of the base? Use a complete sentence in your answer.

What is the height of the speaker cabinet? Use a complete sentence in your answer.

Find the volume of the speaker cabinet. Show all your work and use a complete sentence in your answer.

